Prospective elementary teachers' number sense performance in Taiwan

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Abstract: To examine prospective teachers' number sense performance, two-hundred and eighty prospective teachers who majored in elementary teaching from one teacher training institution of southern Taiwan were selected to participate in this study. Results showed that about one-fifths of the prospective teachers could apply number sense based method (such as using benchmarks or estimation appropriately) and about a half of prospective teachers relied on rule-based methods when responding to number sense questions. Results also indicated that these prospective teachers highly focused on written method to solve the number sense questions. It is reasonable to believe that if we want to improve students' number sense, then an important action that should be taken is to improve the quality of their teachers' knowledge on number sense.

Keywords: Mathematics. Number sense. Prospective teachers.

Rendimiento del sentido numérico de futuros profesores primarios en Taiwán

Resumen: Para examinar el rendimiento numérico de los futuros docentes, se seleccionaron para participar en este estudio doscientos ochenta futuros maestros que se especializaron en enseñanza elemental en una institución de capacitación de maestros en el sur de Taiwán. Los resultados mostraron que aproximadamente una quinta parte de los maestros potenciales podrían aplicar el método basado en el sentido numérico (como el uso de puntos de referencia o la estimación de manera apropiada) y aproximadamente la mitad de los maestros prospectivos se basaron en métodos basados en reglas cuando respondían a preguntas numéricas. Los resultados también indicaron que estos futuros docentes se enfocaron en el método escrito para resolver los problemas de sentido numérico. Es razonable creer que si queremos mejorar el sentido numérico de los estudiantes, entonces una acción importante que se debe tomar es mejorar la calidad del conocimiento de sus maestros sobre el sentido numérico.

Palabras clave: Matemática. Sentido de los numerous. Futuros docentes.

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Desempenho do senso numérico de futuros professores primários em Taiwan

Resumo: Para examinar o desempenho numérico dos futuros professores, duzentos e oitenta desses professores, que se especializaram no ensino fundamental, foram selecionados de uma instituição de treinamentos de professores do Sul de Taiwan para participarem desse estudo. Os resultados mostraram que, aproximadamente, um-quinto dos futuros professores puderam aplicar o método com base no senso numérico (como a utilização de pontos de referência ou a estimativa de maneira apropriada) e, aproximadamente, metade desses professores dependeram de métodos baseados em regras quando responderam as questões sobre sentido numérico. Os resultados também indicaram que esses futuros professores estavam altamente focados no método escrito para resolver as questões de sentido numérico. É razoável acreditar que, se quisermos aperfeiçoar o senso numérico dos alunos, uma ação importante que deve ser tomada, é melhorar a qualidade do conhecimento dos professores sobre o senso numérico.

Palavras-chave: Matemática. Senso numérico. Futuros professores.

1 Introduction

Promoting children's number sense has been internationally thought to be a key issue in mathematics education during the past two decades (ANGHILERI, 2000; DUNPHY, 2007; JORDAN, GLUTTING e RAMINENI, 2010; MCINTOSH *et al.*, 1997; NCTM, 2000; VERSCHAFFEL, GREER e De CORTE, 2007; YANG e LI, 2013).

However, several studies have shown that many children in the elementary and middle grade levels performed poorly in number sense (MARKOVITS e SOWDER, 1994; MCINTOSH, REYS e REYS, 1992; REYS e YANG, 1998, 2005). In addition, Yang (2007) also showed that prospective teachers tended to use written methods to solve numerical problems.

To support children's learning of elementary mathematics meaningfully, elementary teachers need to understand that mathematics deeply and flexibly (MA, 1999; YANG, 2007). In other words, the teacher should have good number sense (REYS e YANG, 1998). However, researchers have found that prospective elementary teachers tend to rely on standard algorithms and reason inflexibly (MA, 1999; YANG, 2007).

Children's lack of number sense may be due to their teachers' insufficient competency to teach number sense. As Ma (1999) suggested that, "to improve mathematics education for students, an important action that should be taken is improving the quality of their teachers' knowledge of school mathematics" (p. 144), teachers should have profound understanding on number sense and know how to teach number sense should be the priority for helping children develop number sense.

Since prospective teachers will be school teachers in the future, their profound understanding on number sense is also very important because teachers' mathematical knowledge will affect their students' learning and understanding (BALL, HILL e BASS, 2005). These points promote the necessity of examining prospective teachers' performance on number sense.

Therefore, the research question guiding the study was: *How is the prospective teachers' number sense performance when responding to number sense- related problems?* Ultimately, the results of this study would provide insights into the phenomenon of prospective elementary teachers' development and understanding of number sense, which will inform revisions and elaboration to the local instruction theory.

2 Background

The emphasis on helping children develop number sense has been widely accepted in mathematics education (NCTM, 2000; VERSCHAFFEL, GREER & DE CORTE, 2007). Although different researchers may define number sense and its components in different ways (BERCH, 2005; VERSCHAFFEL, GREER & DE CORTE, 2007), most of number sense related studies and reports (e.g., BERCH, 2005; MARKOVITS e SOWDER, 1994; MCINTOSH *et al.*, 1997; NCTM, 2000; REYS & YANG, 1998; YANG, 2007) believe that understanding the meanings of numbers, recognizing the relative number magnitude, recognizing the relative effect of operation on numbers, and judging the reasonableness of a computational result are key elements of number sense.

Accordingly, this study defined number sense components for prospective teachers as follows:

(1) Developing and using benchmarks appropriately – It implies a person can use the benchmarks, such as $1, \frac{1}{2}$, and so on, to solve problems flexibly and appropriately under different situations (MCINTOSH *et al.*, 1992). For example, when persons were asked to decide the decimal point $0.4975 \times 9428.8 = 4690828$, they knew that .4975 is about $\frac{1}{2}$ and nine thousands multiplied by $\frac{1}{2}$ should be about four thousands more. Therefore, the result should be 4690.828.

(2) Recognizing the relative number magnitude – It implies a person can recognize the size of numbers. For example, when comparing the fractions $\frac{30}{31}$ and $\frac{36}{37}$, they do not need to rely on the written methods (such as finding the least common denominator suggested in the mathematics curriculum). However, they can use meaningful and flexible way, such as residual strategy (CRAMER, POST and DELMAS, 2002) to compare fractions (e.g., $\frac{30}{31} + \frac{1}{31} = \frac{36}{37} + \frac{1}{37}$ and $\frac{1}{31} > \frac{1}{37}$, so $\frac{30}{31} < \frac{36}{37}$).

(3) Understanding the relative effect of operation on numbers – This means that an individual should recognize how the four basic operations affect the computational results (MCINTOSH *et al.*, 1992). For example, when asking children to find the best estimate for 129×0.96 or $129 \div 0.96$, they should be able to make sense of the meaning of the operations, and understand that multiplication or division is not always to get larger or smaller numbers (GRAEBER and TIROSH, 1990; GREER, 1987).

(4) Judging the reasonableness of a computational result – This implies that individuals can mentally apply estimation strategies to problems without using written computation

(MCINTOSH *et al*., 1992; SOWDER, 1992). At the same time, they also should be able to judge the reasonableness of the result. For example, when asking children to decide the best estimate for $291 + 475 + 574 + 875$, $41719.178 \div 19.295$, 48.775×58.985, or $623.91 \div 0.2499$ is the closest to 2500. They should be able to use the estimation strategy to decide the answer and knew this estimation is reasonableness.

3 Studies related to prospective teachers' subject matter knowledge

The results of Ma's (1999) study suggest that mathematics teachers' subject matter knowledge play a key role in teaching mathematics. NCTM (2000) in *The Principles and Standards for School Mathematics* also emphasizes, "Effective teaching requires knowing and understanding of mathematics." (p. 17).

Several studies (e.g., BALL & BASS, 2003; HILL, SCHILLING & BALL, 2004) argue that mathematics teachers should have specialized mathematical content knowledge when they are teaching mathematics. HILL *et al*. (2007) further state that teachers' mathematical knowledge typically shows a positive relationship to student mathematics achievement. These statements highlight the role of mathematics teacher's subject matter knowledge when teaching mathematics.

Several number sense-related studies and documents suggest that teachers should play an important role in helping children develop number sense (ANGHILERI, 2000; MARKOVITS & SOWDER, 1994; YANG, 2003). Yang (2006) further demonstrates that children can develop better number sense if their teachers have profound understanding on number sense and know how to nurture their students' number sense.

These studies also showed that teachers play a key role in creating a learning environment that provides rich opportunities for children to explore numbers, operations, and their relationships. However, there is a limited study which focused on examining elementary teachers or prospective teachers' number sense. This encouraged us to examine the prospective teachers' number sense performance to serve as an indicator to innovate the teacher education in the future.

4 Method

Participants – 280 prospective teachers from a teacher-training institute of southern Taiwan were selected to participate into this study. They were completing an elementary grades teacher preparation program (in Taiwan, grades 1-6 certification). All of the prospective teachers

need to take two required courses: *Basic Mathematics course* with 3 credit hours and *Mathematics Teaching Method* course with 2 credit hours.

The instrument – Three mathematics educators and three teachers reviewed the number sense items, gave some suggestions, and believed that these items were appropriate for the study. To guarantee that these items were appropriate and the test time for each item was functional, several prospective teachers were interviewed to check the appropriateness of the instrument. Based on the results of interview, two items were revised, and finally the instrument included 12 open-ended questions of the four number sense components, with a total of 3 questions in each component.

Procedure – Each participant was given an examination paper. Each page included one open-ended question, and much empty space was given to encourage the participants to write down their ideas. Participants needed to complete each open-ended question within 3 minutes. Therefore, the test took about 40 minutes, including the total testing times and the administration times.

4.1 Categorization of prospective teachers' responses

The participants' responses were initially examined for correctness. To identify the different methods used by the participants, each response (whether correct or not) was defined based on the following four categories:

Number sense-based (NS-based) method

Strategies utilizing at least one of the four components of number sense are considered as evident. For example, the participants responded:

"(1) $\frac{17}{29} < 1$ and $\frac{6}{13} < \frac{1}{2}$, the result of the multiplication should be smaller."

"(2) $\frac{1}{2} < \frac{17}{29} < 1$ and $\frac{6}{13} < \frac{1}{2}$, and $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, therefore, I think the answer is less than $\frac{1}{2}$."

These responses were coded as "number sense-based methods" because they applied 1 and $\frac{1}{2}$ as benchmarks and knew $\frac{17}{29} < 1$, $\frac{6}{13} < \frac{1}{2}$.

Partially number sense-based (PNS-based) method

Part of the number sense methods and written computation were used in the explanation. For example, the participants responded:

"(1) $\frac{17}{29}$ = 0.6, $\frac{6}{13}$ = 0.4, 0.6 × 0.4 = 0.24, then the answer is less than $\frac{1}{2}$."

This kind of response was coded as "partially number sense-based method" because they explained $\frac{17}{29}$ is about 0.6 and $\frac{6}{13}$ is about 0.4 (partially applied approximation) and used written computation to find 0.6 × 0.4 = 0.24.

Rule-based method

It applied a verbalized rule associated with standard written algorithms but was unable to go beyond the direct application rules in the explanation. For example, if direct computation was used as: $\frac{17}{29}$ × $\frac{6}{13}$ = $\frac{102}{337}$ < $\frac{1}{2}$, then it was coded as "rule-based method".

Wrong explanation

Participant could not provide appropriate explanation.

The incorrect responses were also analyzed as above methods. The participants' answers and explanations were examined and classified by two raters independently. These initial reviews produced categorization agreement on over 90 percent of the prospective teacher's responses. The remaining responses were re-examined and discussed by both raters until complete agreement was reached.

5 Results

5.1 Significant differences between the methods used by the participants

Table 1 summarizes the methods used by participants in items 1 to 12. A simple chi-square test on the number of participants using the NS-based and Rule-based methods in the four sets of questions, 1-3, 4-6, 7-9, and 10-12 was done respectively. The chi-square tests on all items were also done respectively.

In Q4-6 (*recognizing the relative number magnitude*), it was obvious that participants preferred the Rule-based method to the NS-based method (a total of 559:28, p = .000, Chi- square test). Furthermore, the chi-square tests for Q5 and Q6 (a total of 141:16 & 220:12, p = .003 & .000,

Chi-square test) also showed that these participants highly relied on written methods to solve these questions.

Table 1: Prospective Teachers' Responses to Number Sense Questions

Analysis of Strategies/Item	#1	#2	#3	#4	#5	#6	#7	#8	#9	#10	#11	#12
Correct (%)	31	79.6	54.6	91.1	95.4	67.9	60	84.6	11.1	79.3	87.9	37.1
NS-based	69	108	117	0	96	12	71	103	23	60	71	53
Partially NS-based	7	0	0	67	0	0	0	0	0	0	0	16
Rule-based	5	105	0	179	130	155	72	132	1	155	168	24
Wrong explanation	7	10	36	9	41	23	25	2	7	7	7	11
Incorrect (%)	69	20.4	45.4	8.9	4.6	32.1	40	15.4	88.9	20.7	12.1	62.9
Partially NS-based	35	37	0	0	0	0	0	0	0	0	0	36
Rule-based	146	16	58	19	11	65	55	0	43	35	29	115
Wrong explanation	11	4	69	6	2	25	57	43	206	23	5	25

Summary of frequency of responses	Items 1-3		Items 4-6		Items 7-9		Items 10-12	
NS-based (%)	35		12.9		23.5		21.9	
Partially NS-based (%)	9.4		8		0		6.2	
Rule-based (%)	37.9		66.5		36		62.6	
Wrong explanation (%)	17.7		12.6		40.5		9.3	

Source: Prepared by the Author

The chi-square test could not be used to Q4 because no student applies NS-based method to solve this question. However, it is obvious that participants also preferred the Rule-based method to the NS-based method (a total of 198:0). Looking carefully at these items, one can see that participants relied heavily on the written methods to compare the magnitude of fractional related questions. This was due to these participants were taught to use the written methods, such as finding the common denominator or transfer the fractions to decimals, to solve these questions

when they were in school. For example, Q6 was used to examine participants' ability on the use of the relative number size.

Data shows that over three-fourth of these participants also heavily relied on written methods, such as,

"(1) $\frac{5}{9}$ = .55 and $\frac{9}{19}$ = .47, .5 - .47 = .03, .55 - .5 = .05. Since .03 < .05, so $\frac{9}{19}$ is closer to $\frac{1}{2}$."

"(2) $\frac{5}{9}$ - $\frac{1}{2}$ = $\frac{.5}{9}$ = $\frac{1}{18}$, $\frac{1}{2}$ - $\frac{9}{19}$ = $\frac{.5}{9}$ = $\frac{1}{38}$, $\frac{1}{18}$ > $\frac{1}{38}$ "

"(3) Finding the common denominator: $\frac{5}{9}$ = $\frac{190}{342}$, $\frac{9}{19}$ = $\frac{165}{342}$, $\frac{1}{2}$ = $\frac{171}{342}$, so $\frac{9}{19}$ is closer to $\frac{1}{2}$."

"(4) $\frac{5}{9}$ = .55, $\frac{9}{19}$ = .42, .55 - .5 = .05 and .5 - .42 = .08. So $\frac{5}{9}$ is closer to $\frac{1}{2}$."

"(5) $\frac{5}{9}$ - $\frac{9}{19}$ = $\frac{252}{3078}$, $\frac{5}{9}$ - $\frac{1}{2}$ = $\frac{171}{3078}$ "

"(6) 1 - $\frac{5}{9}$ = $\frac{4}{9}$ and 1 - $\frac{9}{19}$ = $\frac{10}{19}$, so A is closer"

In Q7-9 (*understanding the meaning of numbers, operations and their relationships*), the participants preferred the Rule-based method to the NS-based method (a total of 303:197, p = .000, Chi-square test). The results of the Chi-square test showed that participants who used Rule-based method significantly higher than those who used NS-based method when responding to the Q7-9.

Furthermore, the Chi-square test for Q7 and Q9 (a total of 127:71 & 44:23, p = .000 & .010, Chi- square test) show that these participants also highly relied on written methods to solve these questions. However, no significant trend was found in Q8 between the use of the Rule-based and NS-based methods (a total of 132:103, p = .059, Chi-square test).

In Q7, we can see that these participants tended to use written methods to decide the relationships between number and operations. For example, the participant responded:

"103 × 48 = (100 + 3) (50 - 2) = 5000 + 150 – 200 – 6 = 5000 – 56 = 4944; 100 × 50 = 5000, 103 × 50 = 5000 + 150 = 5150, and 100 × 48 = 4800. Since 5000 – 4944 = 56 is the smallest. So the answer is 100 × 50."

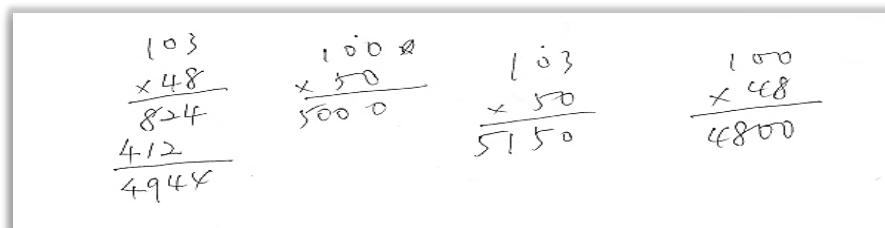Or   "I used the written computation" as following Figure:

Figure 1: Li's Working Practice

Q9 was used to check whether these participants could apply the relative effect of operation on numbers to decide the largest possible product. Data shows that the participants had the lowest correct percentage on Q9. Only 11% of them could exhibit a correct answer and 8% of them could flexibly decide the answer. This is the most difficult question among the other questions. About 89% of the participants incorrectly responded and could not make sense the relationships between numbers and multiplication. For example, they usually explained:

"(1) The answer is 975 × 864, because 9 & 8 should be put in here. Then 7 & 6 should follow 9 & 8 (97_ × 86__). Then 5 & 4 should be put in here (975 × 864)."

"(2) The answer is 954 × 876, because the larger the product, the closer of the two numbers. Since the 954 and 876 are very close, so 954 × 876 should get the largest numbers."

Or "(3) The answer is 945 × 876, because the closer the two numbers, the larger the product."

They misunderstood that two closer 3-digit numbers could get the larger product. Moreover, some of the participants said: "They could not decide the answer without using paper-and-pencil." It seems very difficult for these participants to solve this kind of problem without using paper-and-pencil.

In Q10-12 (*judging the reasonableness of a computational result by using the strategies of estimation*), it was also obvious that participants preferred the Rule-based method to the NS-based method (a total of 526:184, p =.000, Chi-square test). The result of the Chi-square Test showed that participants using Rule-based method significantly higher than those participants using NS-based method when responding to the Q10-12.

The Chi-square tests for Q10, Q11 and Q9 (a total of 190:60, 197:71 & 139:53, p = .000, Chi-square test) show that these participants all highly relied on written methods to solve these questions. For example, in Q11, Over 70% of the participants tried to use written method as follows:

"1500 ÷ 600 = 2.5 and small size 1500 ml need 18 × 2.5 = 45; so the large size is cheaper."

"18 ÷ 600 = .03, 35 ÷ 1500 =.023 < .03, per ml need less money, so large size is cheaper."

"600 ÷ 18 = 33, 1500 ÷ 35 = 42, per dollar can buy 42ml > 33ml, so large size is cheaper.

"Because $\frac{600}{18} \div \frac{1500}{35} = \frac{300}{9} \div \frac{300}{7}$, so the small size is cheaper."

Or "18 ÷ 600 = .03, 35 ÷ 1500 = .023, .03 > .023, so the small size is cheaper."

It is reasonable to believe that written method limits these participants' thinking and hinder the use of number sense.

5.2 No significant difference exists between the strategies used by the participants

In contrast, no significant trend was found in Q1-3 between the use of the Rule-based and NS-based methods (a total of 330:294, p = .161, Chi-square Test). This indicates that there is no difference between the use of Rule-based and NS-based method when participants responded to Q1-3 (*developing and using benchmarks appropriately*). However, in Q1, Rule-based significantly outnumbered NS-based (151:69, p = .000, Chi-square test). It shows that participants who used Rule-based method significantly higher than those participants who used NS-based method when responding to the Q1. Over 50% of the participants tried to use written methods and responded:

"(1) The multiplicand has four decimal points and multiplier has one decimal point, so the result should have five decimal point. Therefore, the answer is (1) 46.90828."

"(2) The answer is 469.0828. Since multiplicand has 4 decimal digits and the multiplier has 1, then the product has 5 decimal digits. However, the last digit of multiplicand and multiplier are 5 and 8.5 × 8 = 40, therefore, one '0' disappeared."

Or responded: "I need to use paper-and-pencil to find the answer."

Since they miscalculated by written methods or wrote the rules, this indicated that these participants could not make sense what they were doing.

Q3 was also used to judge participants' ability on judging the reasonableness of computational results by using the height of one floor about 3 meters as a benchmark. Data showed that about 42% of the participants could use the NS-based method to estimate the correct answer. For example, they responded:

"I think the length, width, and height of the class are about 10m, 10m and 3m, therefore, the volume is about 10 × 10 × 3. So the answer is 300 m^3."

Since they could meaningfully and effectively estimate the length, the width, the height, and the volume of the class, this was a good characteristic of number sense. However, about 45%

of the participants could not come up with the correct answers and estimate the volume of the class. They responded:

"The length = 3m, the width = 5m, and the height = 2m, volume = 3 × 5 × 2 × 30"

"I think the length = 30m, the width = 25, and the height = 3m, so the volume = 30 × 25 × 3 = 2250. So, the answer is about 3000 m³."

"I estimate the length = 30, the width = 30, and the height = 40m, then the volume = 30 × 30 × 40 = 36000, so the answer is (4) 30000 m³."

They could not make sense the length, width and height of the class they situated and this indicated that the future prospective teacher training should connect the activities with real-life situation to enhance their problem solving competence.

6 Discussion

Table 1 also summarizes the frequency of participants' responses on four different components. Several features were observed as follow:

(1) During the test, the participants were not suggested to use paper-and-pencil; however, over one-thirds of the participants relied on written methods to solve questions, either in *developing and using benchmark appropriately* or in *understanding the meaning of numbers, operations and their relationships*. Moreover, about two-thirds of the participants relied on written methods to solve questions, either in *recognizing the relative number magnitude* or in *judging the reasonableness of a computational result*. The finding in relation to participants preferred the Rule-based method to the NS-based method is consistent with the earlier studies of Taiwanese young children (REYS & YANG, 1998, 2005) and international studies (MARKOVITS & SOWDER, 1994).

(2) About one-third of the participants could apply benchmarks such as 1 and $\frac{1}{2}$ to reach appropriate answers. This finding is different from the earlier studies (REYS & YANG, 1998, 2005) that few 6th- or 8th-graders could use benchmarks appropriately to solve problems. The better use of the benchmarks for the prospective teachers might be because they are older and more mature than those young children.

(3) Over a half of the participants preferred to use the method of eliminated impossible answers. It seems difficult for these participants to develop and use the height of a floor which they are situated place as a benchmark to solve Item 3. The lack of this kind of common sense probably because the past training on math is over emphasis on the written computation. Therefore, this

limited their thinking and reasoning. This confirms the statement of Lesh & Lamon (1992) and Organization for Economic Co-operation and Development (2004) that meaningful learning should connect with daily-life situations.

(4) Data showed that the prospective teachers relied heavily on rule-based method when responding to number size items. Over two-thirds of the prospective teachers relied on written algorithms, such as finding the common denominator or changing fractions to decimals, to compare fractional and decimal sizes. Only about one-ninths of them could apply number sense-based methods to solve these questions. This problematic situation was consistent with the results of several earlier studies (e.g., MARVOKITS & SOWDER, 1994; REYS & YANG, 1998, 2005), in which it was found that the 6th- & 8th-graders were highly relied on standard written algorithms to compare fractions and decimals. It is reasonable to assume that to "empower students to think mathematically, teachers must first be so empowered" (MA, 1999, p. 105).

(5) Over 80% of the participants could come up with the correct answer and performed better on *recognizing the relative number magnitude* than other number sense components. This finding also further confirms the earlier studies (REYS & YANG, 1998, 2005) that Taiwanese students have better performance on *recognizing the relative number magnitude* than the other number sense components. However, the methods found in these studies focused on written computation.

(6) The participants are very familiar with the written computation, such as finding the results of 964 × 875, 103 × 48, or 100 × 50; however, it is very difficult for them to apply number sense in solving non-routine problems. This confirms the statement of Markovits & Sowder (1994), "Few students exhibit number sense when solving arithmetic problems in schools" (p. 4). It is also difficult for prospective teachers to apply the relative effect of operation on numbers.

(7) Over 60% of the participants could not apply estimation skills and relied on written computation to decide the answer. This indicated that these participants relied heavily on standard written algorithms; it is difficult for them to estimate a result without using written computation. This finding is consistent with the earlier studies (i.e., REYS & YANG, 1998, 2005) that the use of estimation is a new experience for many Taiwanese students at the 5th-, 6th-, and 8th-grade levels. This indicates that the prospective teachers in Taiwan are also difficult to use the strategy of estimation when solving these questions. At the same time, both the students at the 5th-, 6th-, & 8th-grade levels and prospective teachers seemed to be comfortable finding the exact answer to a problem. This might be due to the topic of estimation does not include in mathematics textbooks.

7 Conclusion

This study focused on number sense and looked beyond an answer to learn about the number sense of 280 prospective teachers in Taiwan. The open-ended questions provided an opportunity to not only better understand their level of number sense but also to explore their thinking of number sense. Although limited to 280 prospective teachers form one University, these data has provided several important and interesting results.

The results reveal that about one-fifths prospective teachers could apply number sense based strategies (e.g., *using benchmarks appropriately* or *recognizing the number magnitude*) and about a half of prospective teachers relied on rule-based methods when responding to number sense questions. This result confirms the statement of Markovits and Sowder (1994), "Few students exhibit number sense when solving arithmetic problems in schools" (p. 4) in the United States and the statement of Yang & Reys (2002) that "Taiwanese students were strongly inclined to use standard written algorithms when explaining their answers" (p. 67).

The de facto tendency of prospective teachers to use written methods seemed to limit their thinking and most likely hindered the development of their number sense. For example, when prospective teachers were asked to compare number size, they often tried to find a common denominator or change fractions to decimals the methods they learned from the textbooks. This finding is also consistent with the earlier studies in Taiwan (REYS & YANG, 1998, 2005) that the 5th-, 6th-, and 8th-graders highly relied on written methods to solve problems.

The low percentage on the use of benchmarks and the high percentage on the use of written methods were similar to the younger peers (5th-, 6th-, and 8th-graders) in Taiwan as found in the earlier studies (REYS & YANG, 1998, 2005). This is due to the mathematics textbooks used in Taiwan usually offered children how to use paper-and-pencil to answer question and never introduce how the benchmarks can be used to help children solve problems.

These problems indicate that the teacher education in Taiwan should put more emphasis on helping prospective teachers develop number sense, such as the use of benchmarks and the use of estimation. Therefore, if we want to improve students' number sense, then we should to improve the quality of their teachers' knowledge on number sense.

This earlier studies and documents (MA, 1999; NCTM, 2000; SCHIFTER, 1999) argue that teachers should have a profound understanding on mathematical knowledge and must "be able to represent mathematics as a coherent and connected enterprise" and "know and understand deeply

the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks" (NCTM, 2000, p. 17).

The findings of this study indicate that, for students to acquire good number sense, teachers must have a profound understanding of number sense and know how to teach number sense. This supports the statement of Ma (1999) that "to empower students with mathematical thinking, teachers should first be empowered" (p. 105).

The findings found in this study suggest that teacher education program should integrate the topic of number sense in the mathematics-teaching course. If significant improvements in mathematics education are to be made, the topic of number sense should become a central focus in teacher education programs.

If we want to help children develop number sense, first of all, we must enhance teachers' number sense. We do hope our findings will encourage more teacher education programs to direct their instruction towards facilitating the prospective teachers' number sense. We also hope that this study will be useful in redirecting any effort for future teacher education programs.

Acknowledgments

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