Planning training spaces and the mobilization of Didactic-Mathematical Knowledge: a look at the Pedagogical Residency Program

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Abstract: This article aims to present a discussion about the planning of training spaces that focus on teaching initiation, considering that these can provide the mobilization of knowledge and, in particular, of Didactic-Mathematical Knowledge (DMK). Therefore, we chose the Pedagogical Residency Program (PRP), as a Public Policy managed by the Coordination for the Improvement of Higher Education Personnel (CAPES/BRASIL) as a training context to support our reflections, which are anchored in the structure proposed by CAPES for the implementation of the PRP and in the composition of the modules (Environment, Semi-structured Observation and Conducting), theoretically based on discussions about the teaching initiation process and on the Ontosemiotic Approach - EOS, especially discussions related to Criteria of Didactic Suitability.

Keywords: Pedagogical Residency Program. Teacher Training Introduction to Teaching. Didactic-Mathematical Knowledge. Criteria of Didactic Suitability.

Planejamento de espaços formativos e a mobilização do Conhecimento Didático-Matemático: um olhar para o Programa Residência Pedagógica

Resumo: O presente artigo tem por objetivo apresentar uma discussão acerca do planejamento de espaços formativos que focalizam a iniciação à docência, considerando que esses podem propiciar a mobilização de conhecimentos e, particularmente, de Conhecimento Didático-Matemático (CDM). Para tanto, elegemos o Programa Residência Pedagógica (PRP), enquanto Política Pública gerida pela Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES/BRASIL) como contexto formativo para fundamentar nossas reflexões, que se alicerçam na estrutura proposta pela CAPES para implementar o PRP e na composição dos módulos (Ambientação, Observação semiestruturada e regência), teoricamente, nas discussões acerca do processo de iniciação à docência e no Enfoque Ontossemiótico - EOS, especialmente, as discussões relacionadas aos Critérios de Adequação Didática.


Planificación de espacios formativos y la movilización del Conocimiento Didáctico-Matemático: una mirada al Programa Residencia Pedagógica

Resumen: El presente artículo tiene por objetivo presentar una discusión sobre la planificación de espacios formativos que focalizan la iniciação à la docencia, una vez que éstos pueden propiciar la movilización de conocimientos y, en particular, de Conocimiento Didáctico-Matemático (CDM). Para ello, elegimos el Programa Residencia Pedagógica (PRP), como Política Pública gestionada por la Coordinación de Aperfeiçoamento de Pessoal de Nível Superior (CAPES/BRASIL), como contexto formativo para fundamentar nuestras reflexiones. Las reflexiones se basan en la estructura propuesta por la CAPES para la implementación del PRP y en la composición de los módulos (Ambientación, Observación semiestructurada y regencia) apoyándose, teóricamente, en las...
discusiones sobre el proceso de iniciación a la docencia y en el Enfoque Ontosemiótico - EOS, en particular, las discusiones relacionadas con los Criterios de Idoneidad Didáctica.


1 Introduction

The formation of Mathematics teachers is under continuous debate nowadays. Curriculum changes in the context of Basic Education and continuous technological movements confirm that licenaties’ spaces are in the center of discussions. Furthermore, in recent years, disbelief in educational processes and, especially, in the figure of the educator, has strained the field of teacher education, placing them in constant alert, apprehension, dispute and struggle.

This scenario is aggravated by the lack of investments, as well as by significant budget cuts. In this sense, as reported by Portuguese educator Boaventura Sousa Santos, in an interview given during the 39th National Meeting of the National Association of Graduate Studies and Research in Education (ANPEd):

Education at all levels, and particularly in higher education, is what sustains, to a large extent, the very vitality of science, not only because the overwhelming majority of research is done in universities, but also because science is carried out by scientists, who have to be educated in good schools, in good universities. Universities that encourage their formation and interest in research. Therefore, at a time when education is the victim of brutal, blind and indiscriminate cuts, it is plain to see that this has a very negative impact on Brazil's entire scientific and educational heritage (SANTOS; OLIVEIRA; SÜSSEKIND, 2019, p. 2).

As a consequence, it becomes absolutely essential, as well as urgent, to promote reflections on the contexts and spaces of initial and ongoing teacher training, in particular, on the contributions that public policies can make to innovation in future teachers' knowledge and practices repertoire. However, for this debate, it is important to recognize that the agents in these spaces, in the most diverse theoretical and methodological aspects, have a lot to add to the discussion and struggle in favor of licentiate spaces and to understanding that teacher education happens in social, economic, political and cultural contexts. Therefore, as highlighted by Curado (2020, p. 111) "(...) teachers are historical subjects, bearers and builders of culture who have the capacity to produce relevant knowledge on education (...)".

Thus, this article aims to present a discussion about the planning of training spaces that focus on teaching initiation, since these can provide the mobilization of knowledge and, in particular, Didactic-Mathematical Knowledge (DMK). Therefore, we chose the Pedagogical Residency Program (PRP), as a Public Policy managed by the Coordination for the Improvement of Higher Education Personnel (CAPES/BRASIL) as a training context to support our reflections. We also sought theoretical support in
discussions about the teaching initiation process and in the Ontosemiotic Approach - EOS, in particular, discussions related to the DMK.

2. The Pedagogical Residency Program

The PRP was created in 2018, although it was not the first public policy focused on the initial training of teachers, for before the PRP, the Institutional Program for Teaching Initiation Scholarships - PIBID already existed. So, the proposal of a new program generated, at the time, a scenario of disputes and tensions, as some professionals claimed that the PRP would “modernize” the PIBID and extinguish it. Tinti and Silva (2020) account that this decision by the MEC caused intense debates and manifestations, since the PIBID was consolidated and had important repercussions among the institutions and agents involved in the formation processes.

After several debates, the PRP was established through Ordinance No. 259, of December 17, 2019, issued by CAPES, assuming the following objectives:

- encourage the training of higher-level teachers for Basic Education, leading the licentiate to actively exercise the relation between theory and professional teaching practice;
- promote the Adaptation of the curricula and pedagogical proposals of the licentiate courses to the guidelines of the Common National Curriculum Base (BNCC);
- strengthen and enhance the relationship between Higher Education Institutions (IES) and public schools of Basic Education for the initial training of Basic Education teachers and;
- strengthen the role of education networks in the future teachers' formation.

To achieve these goals, the following structure for the PRP was proposed:
As can be seen, the PRP seeks articulation between training institutions and public education networks. Such articulation has enabled different repercussions on teacher formation, especially in the field of Mathematics (TINTI; SILVA, 2020), since this area is assumed as one of the priorities within the PRP budget (TINTI; SILVA; FARIA, 2021).

The PRP was implemented from the launch of public notices aimed at selecting proposals, called institutional projects1, presented by Higher Education Institutions (IES) that form teachers from all regions of Brazil. Furthermore, as highlighted by CAPES2, the project to be submitted for consideration must be done in collaboration with the State and/or Municipal Education Secretariats, aiming at articulation with the pedagogical proposal of the partner networks and schools that will receive the teachers.

So far, CAPES has promoted two selections of Institutional Projects, the first in 20183 (250 projects) and the second in 20204 (255 projects). It is important to say that, for each selection, the IES have a period of 18 months to develop the approved Institutional Project. Specifically, the Notice 01/2020, presented a classification in priority and general areas. The priority ones were composed of: Literacy, Biology, Science, Physics, Portuguese Language, Mathematics and Chemistry, while the general ones

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1 An institutional project of the Pedagogical Residency Program consists of subprojects, which correspond to undergraduate courses specified within an institution or in the articulation and integration of two or more areas.
3 Available at: https://www.gov.br/capes/pt-br/centrais-de-conteudo/01032018-edital-6-2018-residencia-pedagogica-pdf
4 Available at: https://www.gov.br/capes/pt-br/centrais-de-conteudo/06012020-edital-1-2020-residencia-pedagogica-pdf
were: Art, Physical Education, Philosophy, Geography, History, Computing, English Language, Spanish Language, Sociology, Indigenous Intercultural, Field Education and Pedagogy.

The dynamics of project implementation takes into account that each area - or the meeting of two or more areas - creates a subproject\(^5\) composed of core segments. The nuclei are made up of a grouping of 08, 16, or 24 future teachers, and it is also possible to have up to 02 (two) volunteers for every 08 (eight) scholarship holders.

![Figure 2: Structuring Institutional Projects](image)

Source: Prepared by the authors.

The PRP participants are named, according to Public Notice 01/2020, as:

- **Resident:** student with active enrollment in a degree course who has attended a minimum of 50% of the course or who is above the 5th period;
- **Preceptor:** teacher of the Basic Education school responsible for planning, monitoring and guiding the Residents in the activities developed in the school-field;
- **Advising Professor:** professor of the Higher Education Institution (HEI) responsible for planning

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\(^5\) In this text, whenever we refer to the development of the Subproject/Mathematics actions within the scope of the partner school, we will use the name Pedagogical Residency Program or the acronym PRP.
and guiding the activities of Residents of their pedagogical residency nucleus, establishing the relation between theory and practice and;

- **Institutional Coordinator**: IES professor responsible for the organization, monitoring and execution of the Institutional Project of Pedagogical Residency.

Regarding its coverage, Tinti, Silva and Faria (2021), in a study based on the result of Public Notice 01/2020, point out that the distribution of quotas (scholarships) for Residents reached all regions, with the Northeast Region having the largest number of scholarship holders (9,768), the Southeast Region in second place (7,824), followed by the South Region (5,784), in fourth place the North Region (3,456) and, finally, the Central-West Region (3,264). In this scenario, 16,680 scholarships were allocated to priority areas and 13,416 to general areas, totaling 30,096 quotas distributed throughout Brazil.

Previous studies have shown that the PRP is an important space for the formation of future Mathematics teachers, in particular, with regard to the approximation between theory and the reality of the professional field (SILVA, 2018; TINTI and SILVA, 2020; SILVA and VIANA, 2020). Moreover, further investigations into the PRP are needed, as evidence shows that this policy of consolidation of teaching initiation practices can be a fruitful space for the development of different aspects of the Mathematics teacher's knowledge.

In view of the discussions and considering that "... the PRP is a relatively new Program and that there are still many issues to be investigated" (TINTI and SILVA, 2020, p. 168), we are proposing, in this article, to approach the PRP as a possible space for the development of the DMK based on reflections on the potential of the Criteria of Didactic Suitability. Given that, in the aforementioned Program, future teachers experience different moments which include processes of environment adaptation, observation of the Basic Education school, planning classes and conducting activities, as discussed in section 5 of this article. This process leads us to the consensus that the act of teaching requires the teacher to make important decisions in the phases of planning, executing and evaluating the process (GODINO, 2013).

3. Introduction to Teaching.

In the scientific field, Initiation into Teaching can be understood as one of the stages of a teacher's formation cycle (HUBERMAN, 1992; LIMA, 2004; GAMA, 2007; TINTI and NAKAYANMA, 2021). Among various perspectives and understandings, Huberman (1992) defends Teaching Initiation as the first stage of the Life Cycle of teachers and corresponds to the first two or three years of professional practice, characterized by aspects of survival and discovery, usually lived in parallel.

In Huberman's view (1992, p. 39) survival is identified as:
the 'clash of the real', the realization of the complexity of the professional situation: the constant groping, the preoccupation with oneself ('can I endure it?'), the distance between the ideals and the daily realities of the classroom, the fragmentation of work, the difficulty in dealing, simultaneously, with the pedagogical relationship and the conveyance of knowledge, the alternation between either too intimate or too distant relationships, difficulties with students who create problems, with inadequate didactic materials, etc.

In turn, the aspect of discovery is what, according to Huberman (1992), allows the teacher to withstand the 'shock with reality', as "the initial enthusiasm, the exaltation at being, finally, in a situation of responsibility (having their own classroom, their own students, their own curriculum), because they feel part of a certain professional body" (HUBERMAN, 1992, p. 39), is replaced by the reality of the classroom, which can prove to be discouraging in many cases.

In order to characterize the process experienced during the first year(s) of the profession, Silva (1997) points out that Müller-Fohbrodt and others (1978 apud SILVA, 1997) used the expressions "reality shock" ("reality shock") and "transition shock".

In order for teachers starting in the profession to learn to manage the dilemmas inherent in their professional activity, without becoming a source of frustration, anxieties or, ultimately, professional lack of motivation, it is necessary for beginning teachers feel the need to create and develop - in line with the characteristics of the school community in which they practice the profession - their own continuing formation project that allows them, through the transformation of their belief system, the improvement of their self-knowledge, their self-esteem and self-concept, become more open to change and develop personally and professionally. (SILVA, 1997, p. 59).

Thus, the initiation to teaching is a complex phase. All the dilemmas experienced by teachers in this period indicate the need to think about Public Policies aimed at helping teachers at the beginning of their careers to develop professionally.

Thus, different countries have specific Teaching Initiation Programs and, it seems to us, that the perspective defended by Huberman (1992), that this process refers to the first years of effective professional practice, guides such proposals, as these, according to the literature, aim to help the beginning teacher to enter the profession, seeking to minimize the effects of the "reality shock":

When someone starts the teaching profession, they fear the lack of Suitability of their ways of thinking and acting with that of their peers; they do not know who to ask for help, nor how to guide their procedures. It's as if, overnight, they suddenly stopped being a student and an increasingly greater professional responsibility fell on their shoulders, for which they seem to be unprepared. (SILVA, 1997, p. 53).

Therefore, these programs take into account a set of factors that directly affect the work to be carried out in the school environment. For this reason, Marcelo García and Vaillant (2017) emphasize that "international experience teaches us what teachers need, in order to learn formative experiences inside
and outside school, observation, feedback, reflection and collaboration [...] (p. 23). It is important to say that, in general, these Programs, which can also be called Induction Programs, offer support and guidance, with a view to promoting learning and the development of the professional knowledge base and helping to socialize these professionals with the school culture. (FERREIRA and REALI, 2005).

However, it is important to highlight that, in Brazil, Teaching Initiation Programs occur in a different way. In some states and cities across the country, there are specific policies to support beginning teachers. One such example is the city of Sobral in the state of Ceará (CALIL and ANDRÉ, 2021), which illustrates specific initiatives that are not linked to a National Policy for Teacher Formation.

Thus, it is important to say that, in Brazil, there is no national Public Policy aimed at teachers who are beginning their careers. What there are the PIBID and PRP Programs that, in a way, seek to contribute to the minimization of the future “reality shock” already in the initial training process, as indicated by Sousa and Gama (2021) when they state that: “[...](p. 139).

In this sense, the PIBID and PRP Programs support and already include undergraduate students in the school context, under the guidance of a teacher from the educational institution and a teacher from the Basic Education school. However, it is important to say that, unlike the experiences of other countries, in Brazil, by entering the aforementioned Programs, teachers in training do not assume all the attributions and responsibilities of professional performance, since the Basic Education teachers who accompany them are responsible for the classes of the schools in which they are inserted.

In particular, regarding the PRP, Tinti and Silva (2020) indicate that this Public Policy seeks the insertion and immersion of future teachers in the professional locus in actions that have as a starting point the real school context, which can corroborate a process of reflection on professional knowledge for teaching.

The aforementioned sets important challenges for the elaboration of Policies and Programs, both for actions aimed at initial training, and for the context of the beginning of a career, in particular, regarding the quality of the pedagogical practices undertaken. Such Public Policies are substantiated by regulations, decrees and public notices, but the teacher trainers are faced with the challenge of planning projects in each area that normally need diagnostic actions, immersion in contexts, conducting classes and evaluating practices.

That said, we must reflect on criteria/pointers/parameters that can support the construction and development of training actions, as teacher training calls us to think about the complexity of building...
knowledge in specific areas, teaching processes and learning, from the instrumentalization of social, political, economic and cultural practices and contexts in which actions are developed.

In this regard, considering that Institutional Projects linked to the PRP may have specific subprojects aimed at future Mathematics teachers (Figure 2), we understand that the DMK perspective can contribute and support the planning and development of actions. Thus, below, we will reflect on the DMK to, later, ponder on its contributions to the structuring and implementation of Mathematics subprojects within the scope of the PRP.

4 Didactic-Mathematical Knowledge - DMK

In this section, we briefly review the teacher training knowledge and then discuss the Criteria of Didactic Suitability in a dialogue with the context of the Pedagogical Residency Program.

The 1980's became a landmark in terms of literature focused on research on teachers' knowledge, as it was when important studies came up to become models and references for research in the field of teacher training until these days. The works by Shulman (1986), Grossman (1990), Mishra and Koehler (2006) and Hill, Ball and Schilling (2008) are examples of discussions that sought to list categories that constitute substantial bases for discussing teacher knowledge in a broader context.

However, other debates were also responsible for discussing teachers' knowledge, but with a specific focus on Mathematics, such as: Ball, Lubienski and Mewborn (2001), Rowland, Huckstep and Thwaites (2005), Llinares and Krainer (2006), Ponte y Chapman (2006), Ball, Thames y Phelps (2008), Hill, Ball y Schilling (2008) and Schoenfeld y Kilpatrick (2008).

More recently, Godino (2009), with a sustained approach in the field of Psychology, Mathematics, Epistemology, Pedagogy, Sociology, Semiotics and Mathematics Didactics, undertook the discussion on “Didactic-Mathematical Knowledge - DMK”. For the aforementioned author, the knowledge of the teacher who teaches mathematics bears complexity and lacks reflections that allow more detailed analyzes of the practices.

The term DMK comes from the context of the “Ontosemiotic Approach to Mathematics Knowledge and Instruction” (EOS) (GODINO, CONTRERAS and FONT, 2006; GODINO, BATANERO, FONT, 2007;2008). For Godino, Batanero and Font (2008), the EOS is a theoretical framework that aims to link different perspectives and theoretical notions, addressing mathematical knowledge and its teaching and learning process in a global manner.

Increasingly, the EOS occupies a prominent place in discussions in the field of Mathematics Education. It is possible to find relevant publications of articles in journals, book chapters and master's
and doctoral theses. We highlight the special thematic section published in Revemop Journal\(^6\), in 2021, which brought together important approaches to EOS. Furthermore, this theoretical construct has\(^7\) its own repository with the availability of categorized studies.

Discussions about the knowledge of Mathematics teachers have sought to point out specific components and indicators that can support the practices of planning, executing and evaluating the teaching and learning processes of mathematical content (GODINO, 2009; PINO-FAN, FONT and GODINO, 2013; PINO-FAN and GODINO, 2015).

In his approach, Godino (2009) proposes a set of facets that are organized into categories, as illustrated in Figure 3.

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\(^6\) Thematic Section organized by the Professors: Dr. José Fernandes da Silva (Universidade Federal de Ouro Preto, Dr. Douglas Silva Tinti (Universidade Federal de Ouro Preto) e Dr. Luis Pino-Fan (Universidad de Los Lagos – Chile). Available at: https://periodicos.ufop.br/revemop. Accessed on Nov. 27 of 2021.

\(^7\) Repository of studies and research related to EOS. Available at: http://enfoqueontosemiotico.ugr.es/. Accessed on Nov. 27 of 2021.
In summary:

The aforementioned facets must be analyzed through the practices that constitute the actions to solve the mathematical tasks and the configurations of the objects and meanings involved, the norms and metanorms that seek to identify the set of rules and habits that condition and make possible the process.
of study. In this context, “a subject is considered to understand a certain mathematical object when she or he uses it competently in different practices” (RODRÍGUEZ-NIETO et al., 2021, p. 8). And, finally, the Suitability, which is considered important to identify potential improvements in the teaching and learning process of Mathematics (GODINO, 2009; BREDA, BOLONDI and SILVA, 2021; BRAGA and SANTOS-WAGNER, 2021).

Still, with respect to didactic Suitability, it is important to highlight its use to identify potential improvements in the teaching and learning process of Mathematics. In other words, anchored in the study by Breda, Font and Lima (2015), we understand that the concept of Suitability must be understood as a set of indicators that establish how a teaching and learning process should be carried out.

Thus, it is necessary to consider the notion of didactic suitability as a possibility to constantly reflect on the improvement of the teaching and learning process.

What criteria should be used to design a sequence of tasks to assess and develop students' mathematical competence and what changes should be made in their redesign to improve the development of this competence? The criteria of suitability (CI) can serve first to guide the teaching and learning processes of mathematics and, secondly, to evaluate its implementations. The criteria of suitability are useful in two stages of the instruction process. A priori, the criteria of suitability are principles that guide "how things should be done". A posteriori, the criteria are used to assess the teaching and learning process effectively implemented. (SECKEL; BREDA; FONT, 2019, p. 440).

Yet, we must highlight that, in addition to using the notion of didactic Suitability in the teaching and learning process, it can be useful to analyze "... curriculum programs, textbooks, student production, among other aspects, both partial and global" (MALET; GIACOMONE; REPETO, 2021, p. 3).

For Godino (2013), the theoretical construct of Didactic Suitability was defined as a didactic that is oriented towards effective intervention in the classroom. For this purpose, six dimensions were considered: Epistemic, Ecological, Cognitive, Affective, Interactional and Mediational. In addition, according to the aforementioned author and Breda, Font and Pino-Fan (2018):

- **Epistemic Suitability** is related to the representativeness of the prescribed curricula and the quality of the mathematics taught;
- **Cognitive Suitability**, expresses the degree to which the intended/implemented meanings are in the students' potential development zone, as well as the observance of advances and limits of student learning;
- **Interactional Suitability**, regards the configurations and trajectories that allow, on the one hand, identifying potential semiotic conflicts; and on the other hand, allow for the resolution of conflicts that occur during the instruction process;
- **Mediational Suitability**, degree of availability and Suitability of material and temporal resources necessary for the development of the teaching process;
- **Affective Suitability**, degree of involvement (interest, motivation, ...) of students in the study process. Affective Suitability concerns both factors that depend on the institution and factors that basically depend on the student and his/her previous school history;
- **Ecological Suitability**, degree of conformity of the study process with the education project of the center, school and society, as well as the social, political, cultural and economic contexts.

In summary:

**Figure 5: Criteria of Didactic Suitability**

To determine the Suitability levels, it is necessary to have clear and explicit guidelines on the objectives and general lines of action (GODINO, 2013). Thus, the indicators of the different adaptations can constitute a guide for the design and evaluation of planned or effectively implemented training actions.

**5 Possibilities for mobilizing Criteria of Didactic Suitability in the Teaching Initiation process**

According to item 3.2.1 of Public Notice 01/2020, students with active enrollment in a degree course who have attended a minimum of 50% of the course or who are studying from the 5th period onwards are eligible to participate as PRP residents. We understand that this cut for participation in the PRP, as well as the actions to be undertaken throughout the modules, shows that the target audience...
needs to present an academic maturity, as well as consolidated learning of the set of knowledge necessary for teaching.

Upon being included in the PRP, future teachers develop actions in partner schools, under the guidance of the preceptor, which include the axes of environment, semi-structured observation and mastery, as illustrated in Figure 6. In addition, as indicated in the aforementioned notice, each of the 3 modules of 138 hours must include the following activities:

- **Environment**: experiencing the school routine to get to know the functioning of the school and the organizational culture, monitoring pedagogical planning activities, identifying how the school is articulated with families and the community, among other aspects;

- **Semi-structured observation**: observation in the classroom based on a script defined by the resident together with the supervisor and;

- **Mastery**: prepare lesson plans and deliver content in the classroom or thematic workshops at school, with the preceptor's follow-up.

The Notice also indicates the following distribution of hours:

- **86 hours of team preparation**, study on the contents of the area and on teaching methodologies, familiarization with the teaching activity through the school environment and semi-structured observation in the classroom, preparation of the resident's report together with the preceptor and the supervisor teacher, evaluation of experience, among other activities;

- **12 hours of lesson plan elaboration** and;

- **40 hours of mastery** with preceptor's follow-up.

It is worth mentioning the number of hours dedicated to “Environment and Semi-structured Observation”, which bring together the preparation activities of the PRP team (Counseling Teachers, Preceptors, Residents) foreseen in 86 hours of each module. In summary, the structure of each module needs to include:
Given this structure of the PRP, we understand that some questions are important to reflect the practices of future teachers in this context:

- How can modules be designed to mobilize mathematical knowledge for teaching? What is the role of trainers (Counseling Teacher and Preceptor) in this context?
- What methodological choices can be made?
- How to select the best resources for educational practices?
- What technological resources can and should be used?
- How to deal with the different contexts of Basic Education schools?
- What reflections and interventions can be made in view of the diversity of learning levels of Basic Education students?
- How to make use of social and cultural reality to enrich pedagogical practices?
- How can future teachers develop reflective practices in the context of PRP?

In an attempt to ponder on these questions, we sought the components and indicators of Didactic Suitability for the debate and promoted a close dialogue between these and the guidelines of the Pedagogical Residency Program. However, considering the breadth of the PRP, we take for reflection general aspects of this public policy, particularly issues related to the planning of actions to be undertaken by trainers and the reflective nature that they need to mobilize in order to implement the Institutional Project. To this end, we organized the discussion into three axes, in order to show how the structure of the PRP can favor the mobilization of the DMK through the Criteria of Didactic Suitability.
a) Insertion in the School Context and Mathematical Content studies.

Upon being included in the PRP, Residents are accompanied by a teacher from the partner school, called preceptor, and by a professor from the university, called counseling teacher. The preceptor can monitor up to 10 residents in the school environment and, therefore, this dynamic demands an organization and distribution of scholarship holders in order to serve the different classrooms present in the institution. The organization of the PRP/subproject within the partner school requires from the preceptor the autonomy to make decisions that are present and inherent in the educational process.

As an example, we can have a partner school that only attends the final years of Elementary School (6th to 9th grade - teenagers from 11 to 15 years old), or High School (1st to 3rd year - Youngsters from 15 to 18 years old of age), or that meets both. In addition, we can have schools that work in different modalities (Regular, Youth and Adult Education, Professional and Technological Education) among others.

The described scenario requires from the preceptor the ability to liaise with managers, pedagogical support and teachers in the same area (in the case of Mathematics), aiming to distribute residents in an equitable and democratic manner that meets the school demand in relation to the teaching and learning process. In this way, the preceptor may choose to distribute all Residents in classrooms composed only of students of a single year (e.g., 6th year) or in order to meet different years and educational levels.

Thus, when taking into account the needs of the partner school, the preceptor's decision-making, for one or another configuration of distribution of Residents to serve the institution's students, implies carrying out a study and planning process focusing on the prescribed mathematical contents in the
curriculum. For example, in order to serve 6th grade students, it is necessary for Residents to undertake studies on Numbers and their Operations, Geometry, Magnitudes and Measures, Reading and Interpreting Graphs and Tables, among others.

As can be seen, the curricular, institutional, social, political, economic and cultural contexts - *Ecological Suitability* - are present in the insertion phase of future teachers in the context of schools, as these, historically, represent society, which naturally is multifaceted. Thus, reflection on the current curriculum is a necessary stance in the context of teacher education, as there is no curricular neutrality. On the contrary, we agree with Silva (2003):

> The curriculum is place, space, territory. The curriculum is a power relation. The curriculum is trajectory, journey, course. The curriculum is autobiography, our life, curriculum vitae: identity is forged in the curriculum. The curriculum is text, discourse, document. The curriculum is an identity document. (SILVA, 2003, p.150).

In this way, substantiated by different reflections about the context of the partner school, it is expected that future mathematics teachers have the basic skills to mobilize problem-situations, mathematical language, concepts, formalizations, as well as the conditions to foster the understanding of mathematical rules and standards – *Epistemic Suitability* - (GODINO, 2009; 2011). Therefore:

> The teacher, besides the mathematics that allow her or him to solve problems in which she or he mobilizes her / his common and expanded knowledge, must have a certain amount of mathematical knowledge "profiled" for teaching; that is, the teacher must be able to mobilize different representations of a mathematical object, solve the task through different procedures, link the mathematical object to other mathematical objects at the educational level at which it is taught or from previous and subsequent levels, understand and mobilize the diversity of partial meanings for the same mathematical object (which integrate the holistic meaning for this purpose), provide various justifications and arguments and identify the knowledge brought into play during the resolution of a mathematical task. (PINOFAN and GODINO, 2015, p. 99).

In addition to the aforesaid, it is up to the teacher to have an investigative and reflective attitude in the process of developing mathematical concepts.

> Besides their solution, is there another way to solve the task? How would you explain the solution of the task to a student who could not solve it according to the procedures seen in the class? What knowledge is at stake in solving the task? (PINOFAN and GODINO, 2015, p. 99).

However, it is necessary to reflect the planning process of educational activities in Mathematics. The specific knowledge of the area, in the case of Mathematics, is essential, although it is not enough for the contemporary classroom. The Mathematics teacher has the expertise to plan, choose the best resources, reflect on the best approaches to the content, seeking subsidies in the different theoretical-
methodological approaches - *Mediational Suitability* - that are present in the field of Mathematics Education.

One of the fundamental steps in the PRP participation process is the elaboration of lesson plans for the dedication of 40 hours of mastery, followed up by the preceptor. Therefore, the future teacher must have a vast repertoire of knowledge about the contents of the area, in particular, the elaboration and validation of conjectures (MARKIEWICZ and ETCHEGARAY, 2021). This perspective is closely linked to the notion of *Epistemic Suitability* proposed by Godino (2011), which is related to the mathematical knowledge involved in the educational context and its organization for the teaching process.

*b) Study of Mathematics Teaching Methodologies*

Concomitant to the process of studying the mathematical content referring to the context in which they will work, both trainers and residents need to reflect on the main theoretical-methodological approaches that will guide the teaching and learning process. It is worth highlighting the multiplicity of theoretical approaches, especially in the field of Mathematics Education, which seek to trigger reflections on the approach to the contents prescribed in Basic Education curricula.

Returning to the example mentioned in item "a", if residents work in the 6th year of Elementary School, they will need to study/know different theoretical and methodological perspectives that can support the approach to Numbers and their Operations, Geometry, Magnitudes and Measures, Reading and Interpreting Graphs and Tables on other topics that make up the curriculum for this stage of schooling. It is expected that, at this points, debates arise on Semiotic Representation, Ethnomathematics, Mathematical Modeling, Problem Solving, Critical Mathematics Education, History of Mathematics, History of Mathematics Teaching, Financial Education, Statistical Education, Theory of Conceptual Fields, Reading and Writing in Mathematics, Philosophy of Mathematics, Inclusive Mathematics Education, Digital Technologies and Mathematics Teaching, Gender Relations and Mathematics and Ontosemiotic Approach, among others.

We understand that, from this perspective, training actions fall into aspects that relate to the universe of cognitive knowledge, emotions, interactions, the instrumentalization of pedagogical practice, as establishing the relations between theory and practice requires reflecting on specialized knowledge for teaching (LUZ and ALENCAR, 2020). Thus, the relation between theory and practice is one of the pillars of the PRP.

Undoubtedly, educational practice requires that teachers get to know their students better, because, with reflection and evaluation, it is possible, from the point of view of the educational institution, to monitor the learning process - *Cognitive Suitability* - (GODINO, 2011; SILVA et al. MANRIQUE, 2021).
Besides, it is worth noting that the initiation to teaching is a key moment for future teachers to reflect on the importance of the students' prior knowledge, the need for curricular adaptations, promotion of good learning and the use of evaluation as a training tool (GODINO, 2021).

Furthermore, it is important to point out that the PRP perspective of putting the future teacher in contact with the Basic Education classroom presents the challenge of issues related to the affective universe of teaching, as the teaching and learning process is surrounded by related elements, attitudes, emotions, beliefs and values – *Affective Suitability*. In this regard, the practices undertaken within the scope of the PRP need to take into account whether the mathematical tasks are interesting for the students, if they promote participation, as well as attitudes of empathy, perseverance, in addition to involving respect for emotions such as self-esteem, beliefs, values, thus avoiding fear-based attitudes and difficulties with mathematics (GODINO, 2011).

Yet, it is necessary to consider the need for the teaching initiation process to foster future teachers' ability to implement their own practice (SILVA, PIETROPAOLO, FONT, 2017). This means that, when preparing for the phase of mastery classes, the agents involved have reflected and - continue to reflect - on how to better articulate materials and technologies for teaching. In this regard, it is important to highlight Godino's (2011) propositions concerning the *Mediational Suitability* of a Mathematics teaching and learning process, that is, to what extent the selected resources contribute to introduce good problem situations, acquisition of mathematical language, development of arguments, contextualization, models and visualizations.

Finally, it should be noted that the strengthening of the training process of future teachers also involves understanding the importance of interactions in the Mathematics teaching and learning process. Therefore, it is essential that in the mastery practices within the scope of the PRP, the interactions between future teachers-students of Basic Education, future teachers-future teachers, future teachers-institutions, students of Basic Education-institutions, students of Basic Education-curricular materials and future teachers-curricular materials – *Interactional Suitability* – have a prominent place in the educational process.

So, according to Godino (2011), a clear notion of *Interactional Suitability* can support the communication of content, resolution of various conflicts that may arise throughout the teaching and learning process, seek consensus, foster dialogue and communication between the agents involved and develop aspects of students' autonomy to undertake the construction of mathematical concepts.
The role of trainers (Counseling Teachers and Preceptors) in the environment process must be highlighted, in particular, during the planning and mastery phase, as these are moments in which they demand a very high degree of knowledge and decision-making about the teaching and learning process.

Thus, the movement of studies of different methodologies for teaching Mathematics is an important and necessary training process for the PRP. Given that future teachers master classes, the act of collaborative planning (TINTI and SILVA, 2020) results in an opportunity to bring the prescribed curriculum and the reality of partner schools into dialogue.

c) reflective writing of practice

The movement of writing about the practice is provided for in the legal constitution of the PRP in the form of partial and/or final reports. Final reports are mandatory in order to record the practices experienced, the challenges and constraints found in the training process.

Each agent participating in the PRP has responsibilities for the records of the actions taken, as these and other data, requested by Capes, may serve as proof of compliance with the actions for the purposes of accountability. As a result, the Institutional Coordinator must forward information and reports requested by Capes. On the other hand, the Counseling Teachers must periodically evaluate the residents and issue, together with the preceptor, their performance report, as well as submitting periodic reports to the Project Institutional Coordinator containing description, analysis and evaluation of the activities of the coordinating nucleus. In turn, the Preceptors are responsible for submitting periodic reports to the Counseling teachers that contain the description, analysis and evaluation of the activities of the residents under their guidance. Finally, Residents are asked to record their pedagogical residency activities in reports, in the experience report format.

It is important to point out that writing about the practice should not be understood as a bureaucratic procedure. On the contrary, it is an opportunity for Residents and all PRP agents to develop reflective competence about the experienced process. However, teacher trainers should lead the process of developing reflection on practice (SECKEL and FONT, 2015; 2019) along with future teachers. This perspective leads us to point out that the process of selection of Counseling Teachers and Preceptors must take into account the appropriate profile to act as a teacher trainer, as within the scope of the PRP the actions are not merely "do for the sake of doing"; they are instead a reflective, critical and emancipatory kind of “doing”. It is an act that reminds us of Paulo Freire when he states that teaching is:

Methodical rigor, research, respect for the knowledge of students, criticality, ethics and aesthetics, embodying words by example, taking risks, accepting the new, rejecting any form
of discrimination, critical reflection on the practice, recognition and assumption of cultural identity, having awareness of incompleteness, recognizing oneself as a conditioned being, having respect for the autonomy of the being who is being educated, common sense, humbleness, tolerance, conviction that changing is possible, curiosity and professional competence (FREIRE, 1996, p.14).

In the described scenario, the role of Preceptors takes on a very special meaning, for in addition to teaching Basic Education, they begin to exercise the role of counseling teacher and not mere “supervisors” as indicated in the PRP selection notices. That said, we understand that the PRP, in addition to being a space for initial training, can also be a space for ongoing training and trainer's formation, in which collaborative actions can be developed to encourage innovation, professional ethics, creativity, inventiveness, peer interaction and the ability to reflect contemporaneity.

6 Final Considerations

This article intended to present a discussion about the planning of training spaces that focus on teaching initiation, since these can provide the mobilization of knowledge and, in particular, Didactic-Mathematical Knowledge (DMK).

Considering that, at the time of elaboration of the Institutional Project, the counseling professor works directly with the Institutional Coordinator towards the definition of the concept of teacher formation, as well as in the design of actions to be undertaken, we understand that they have the opportunity to structure a proposal for mobilizing the DMK, given the characteristics that we explore in this article.

Since the PRP modules include actions of insertion in the school context, studies of mathematical content, mathematics teaching methodologies and reflective writing of practice, it was possible to show that the planning of each of the moments can be supported by the Criteria of Didactic Suitability.

Regarding the process of resident's environment adaptation in the context of partner schools, the importance of the role of Preceptors and Counseling Teachers was highlighted. These agents, in a collaborative partnership, can develop training actions that, starting with the diagnosis of reality, can end up in studies and reflections on the proposed curriculum and the context in which the partner school is inserted, as well as on the best ways to address the contents of Mathematics.

Thus, we emphasize that at the time of insertion in the school context, it is necessary to develop actions that are consistent with the assumptions of ecological and epistemic didactic adequations. Awareness of the reality context in which the teaching initiation actions are developed is essential so that future teachers can learn and reflect on the future professional field and, along with this fact, it is necessary to raise awareness on the importance of solid mathematical knowledge. The mastery activities should be
seen by the PRP agents as a moment of great responsibility and commitment to science - in this case Mathematics - and, consequently, to Basic Education students’ learning.

Regarding the studies on methodologies for teaching Mathematics, our analyzes also show the importance of the Criteria of Didactic Suitability, as their indicators enable the planning of actions that can promote the development of the DMK in a clearer and more objective way. Specifically, Preceptors and Counseling Teachers can take the approach of Criteria of Didactic Suitability as support for the planning and organization phase of actions with Residents, which can support the choice of resources for pedagogical practices, the relations established in the teaching and learning process, aspects related to emotions and the understanding of the evaluation as a reflection element to guide and reorient trajectories.

We believe that the EOS approaches, in particular the Criteria of Didactic Suitability, bring important contributions to the teacher education processes. With regard to initial training, these can underpin the construction of the mathematical knowledge base necessary for the undertaking of teaching professionalization. In reference to ongoing formation, we defend the importance of teacher trainers to appropriate the notion of didactic Suitability with a view to (re)elaborating the repertoire of knowledge required for promoting innovative training spaces for future teachers.

We add the importance of the PRP, in the figure of its trainer teachers (Preceptors and Counseling Teachers), to encourage Residents to develop reflective reading and writing about practice. As for reading, we understand that Residents need to become aware of the main theorists in the field of Mathematics Education so that they can reflect on the process of planning the teaching the mathematical contents of the classes in which they undertake the mastery. In relation to reflective writing about practice, this is a fundamental element in teaching, that is, it is an essential ability to record, reflect, communicate and debate the practices carried out.

Finally, it is clear that the development of public policies in the field of mathematics teacher education needs to be supported by a critical and reflective “doing”, which demands theoretical and methodological consistency. The EOS can be a theoretical base repertoire for the planning, execution and evaluation of training actions in the field of initial and ongoing formation for Mathematics teachers.
7. References

BALL, D. L.; THAMES, M. H.; PHELPS, G. Content knowledge for teaching. What makes it special? 


FERREIRA, L. A.; REALI, A. M. M. R. Aprendendo a ensinar e a ser professor: contribuições e desafios
de um programa de Iniciação à Docência para professores de educação física. Anais ... 28ª Reunião Anual da ANPEd, 2005.


GODINO, J. D. Indicadores de la idoneidad didáctica de procesos de enseñanza y aprendizaje de las matemáticas. XIII CIAEM-IACME, Recife, Brasil, 2011.


