Articles Section
2024, Volume 6, e2024002
Pages 1–12
10.33532/revemop.e2024002

revemop
ISSN 2596-0245

# Possibilities in teaching planning to make mathematical knowledge accessible to students with autism spectrum disorder

## Possibilidades no planejamento do ensino para tornar o saber matemático acessível a alunos com Transtorno do Espectro Autista

## Posibilidades en la planificación docente para hacer accesible el conocimiento matemático a estudiantes con Trastorno del Espectro Autista

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### Abstract
This article presents part of the analysis conducted in a study to indicate and substantiate possibilities for planning mathematics teaching in the initial years of basic education for students with autism spectrum disorder. Engeström's activity theory guided the investigation as a theoretical and methodological tool, directing data organization and the design of the analyses. This qualitative study had activities prepared by teachers as its data source. The research revealed that, for the teaching process to effectively contribute to mathematics learning by students with autism spectrum disorder, elements that allow the planning of teaching activities based on the construction of meanings of the mathematical objects studied and the development of skills that precede them must be considered.
**Keywords:** Teaching planning. Activity theory. Mathematics. Autism.

### Resumo
Este artigo teve como principal objetivo desvelar contribuições que atividades voltadas à Educação Financeira a partir do tema inflação trouxeram para novas leituras e escritas do mundo de estudantes do Ensino Médio, em uma perspectiva crítica. Os participantes foram estudantes de uma turma do terceiro ano do Ensino Médio de uma escola estadual do interior de Minas Gerais. No total, foram realizados dez encontros, nos quais o tema inflação foi explorado de várias formas. Para este artigo, escolhemos um deles, no qual examinamos cálculos relacionados à inflação; e reflexões em torno dos impactos desta para a sociedade e possíveis soluções para eles. Os resultados evidenciaram que os estudantes passaram a realizar novas leituras do mundo, em um processo de *empowerment*, que possibilitou ensaios de escritas do mundo.
**Palavras-chave:** Planejamento do ensino. Teoria da Atividade. Matemática. Autista

### Resumen
El objetivo de este artículo es presentar parte de los análisis realizados a través de un estudio con la propuesta de indicar y fundamentar las posibilidades de planificación de la enseñanza de las Matemáticas en los años iniciales de la Educación Básica, para estudiantes con Trastorno del Espectro Autista. La Teoría de la Actividad de Engeström guió la investigación, como herramienta teórica y metodológica, dirigiendo la organización de los datos y el diseño de los análisis. Este estudio cualitativo tuvo como fuente de datos actividades preparadas por docentes. Concluimos que, para que el proceso de enseñanza contribuya de manera efectiva al aprendizaje de Matemáticas por parte de estudiantes con Trastorno del Espectro Autista, es necesario considerar elementos que permitan la planificación de actividades docentes basadas en la construcción de significados de los objetos matemáticos. estudiados y el desarrollo de habilidades que los preceden.
**Palabras clave:** Planificación Docente. Teoría de la actividad. Matemáticas. Autista.

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## 1. Introduction

Given that the current scenario is shaping up to build an inclusive school perspective, we recognize the importance of offering effective teaching of academic content covered in the classroom for target students in special education; thus, we look for answers to the questions that emerge in this context.

Based on the above, we considered a small part of that landscape: mathematics teaching to students with autism spectrum disorder (ASD). For such, we conducted a study to understand teaching elements that can contribute to developing mathematical skills in students with ASD (TAKINAGA, 2015; TAKINAGA; MANRIQUE, 2018). Motivated by the research findings, we are developing another study to present new elements that can contribute to the scientific and school community through didactic-pedagogical proposals that may facilitate the mathematics learning of students with ASD.

Our first research focused on analyzing the activities developed by teachers who teach mathematics to students with ASD, aiming to cover from the initial work, which precedes the teaching of arithmetic and prepares for the construction of the meaning of numbers, to the execution of the addition operation, with representation in mathematical language.

However, for this research, we aim to present part of the data collected from the previous study, which led us to conclude that, for students with ASD to learn mathematics effectively, the teaching process must consider elements that allow teachers to plan teaching activities based on the construction of meanings of the mathematical objects studied and the development of skills that precede their teaching.

To indicate and substantiate the possibilities for planning teaching and developing skills –such as comparisons, classifications, seriations, and inclusions, skills that precede the teaching of mathematical objects "numbers," "operations," and "algebra"– in the following topics, we briefly present the theoretical and methodological framework of the analyses. Then, we describe the activities and their planning, and after that, we present the analyses. Next, we conclude with our considerations.

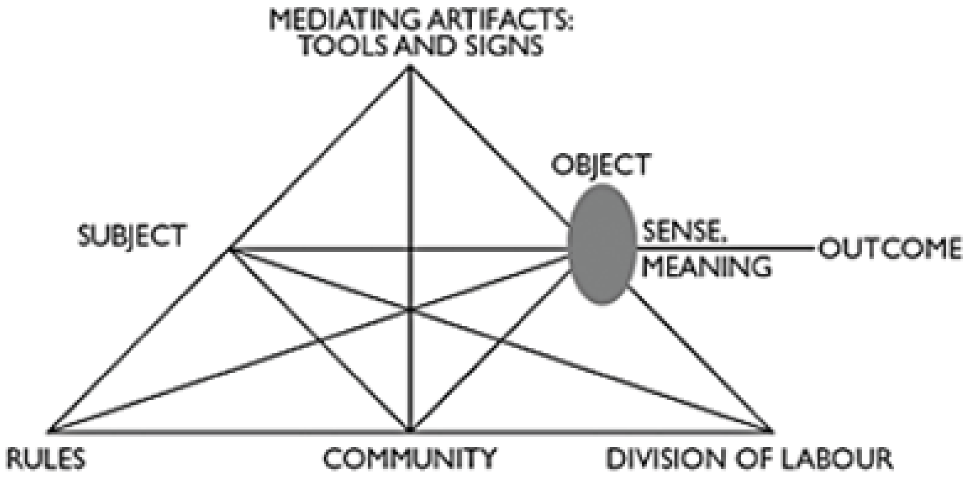
This study was approved by the Ethics Committee on August 13, 2014, registered with protocol number 783.677.

## 2. Theoretical and methodological procedures

Engeström's activity theory guided the investigation as a theoretical and methodological tool, directing data organization and the design of the analyses. This theoretical framework evolved through three generations of research by Engeström (1996 *apud* ENGESTRÖM, 2001). As a theoretical current, activity theory arises from Vygotsky's first-generation studies, and its main contribution is the concept of mediation.

The data was organized through the units of analysis proposed by the theory, conceptualized as activity systems, and their components: subject, mediating artifacts, object, outcome, rules, community, and division of labor. Figure 1 presents the triangular model for an activity system.

Figure 1: Triangular model for an activity system




Source: Engeström (2001, p. 135)

This qualitative study had activities prepared by teachers as its data source. The instruments for data collection were interviews, filming, and monitoring teachers while carrying out the activities. The teachers selected should be teachers of students with ASD, teaching mathematics in the initial years of basic education, and ground their practice on methodological procedures.

For data analysis, we list below the five principles proposed by the theory, which provided the necessary lens so that the activities highlighted the elements that could contribute to the teaching practice of mathematics to students with ASD:

1) A collective, artifact-mediated, object-oriented activity system, seen in its network relations with other *activity systems, is considered the basic unit of analysis*. Goal-directed individual and group actions and automatic operations are relatively independent but subordinate units of analysis, eventually understandable only when interpreted against the background of entire activity systems. Activity systems are executed and reproduced by generating actions and operations.

2) There is a *multiplicity of voices* of activity systems. An activity system is always a community of multiple viewpoints, traditions, and interests. The division of labor in an activity creates different positions for the participants; they drive their own diverse stories, and the activity system, per se, drives multiple layers and threads of diverse stories, and the activity system, per se, drives multiple layers and threads of stories carved by its artifacts, rules, and conventions. The multiple voices in systems networks are multiplied in interacting activity system networks. They are a source of problems and innovations, requiring translation and negotiation actions.

3) A *historicity*. Activity systems take shape and transform over long periods. Their problems and potential can only be understood against their own history. History itself must be studied as the local history of the activity and its objects and as the history of the theoretical ideas and instruments that shaped it.

4) The *contradictions* play a central role as sources of change and development. Contradictions are not the same as problems or conflicts; they are structural tensions that accumulate historically within and between activity systems. Contradictions generate disturbances and conflicts but also create innovative attempts to change the activity.

5) Possibility of *expansive transformations* in activity systems. Activity systems advance in relatively long cycles of qualitative transformations. As the contradictions of a system of activities are compounded, some individual participants begin to question and deviate from their established norms. Sometimes, this becomes cooperative imagination and a deliberate collective effort to change. An expansive transformation occurs when the object and motive of activity are reconceptualized to comprise a radically broader horizon of possibilities than in the previous mode of activity. A complex cycle of expansive transformation can be understood as a collective journey through the zone of proximal development of the activity. (ENGESTRÖM, 2013, p. 72-73)

We will now describe the planning and analysis of the activity through the lens of the embraced theoretical framework. The activity is described and structured in an activity system and subsequently analyzed when we highlight the elements of the teaching and learning process that enhance the development of mathematical skills of students with ASD.

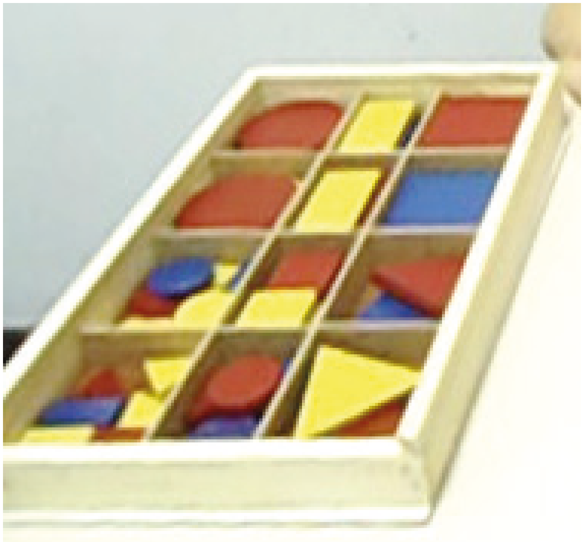
## 3. Teaching planning and activity analysis

The activities described aim to develop logical reasoning and creativity. They are aimed at students with ASD –verbal or non-verbal, as there are people with ASD who can differentiate shapes and colors even without naming them. The activities were planned and organized to encourage comparisons, classifications, and seriations, making these skills –which precede the study of the object “numbers, operations, and algebra” – accessible to students with ASD.

For this purpose, we used the concrete material of logical blocks. This material consists of 48 large, small, thin, and thick pieces in triangle, circle, rectangle, and square shapes in different primary colors —blue, red, and yellow. Through the pieces that make up this material, we can develop manipulative activities with, on the one hand, the straightforward objective of classifying, seriating, and establishing correspondences and, on the other, the indirect objective of preparing for mathematics learning, developing motor coordination, attention, and memory, and organizing logical reasoning, among other skills.

The activities differ in the number of parts used, which gradually increases the possibilities of combining their attributes and requires greater complexity of reasoning for their classification and ordering. For Activity 1, we used 12 pieces of the logic blocks; for Activity 2, 24 pieces; and for Activity 3, 48 pieces. Figure 2 presents the material in question.

**Figure 2:** Logic Blocks Material



**Source:** The authors.

Chart 1 describes the material used in each activity. The three activities use logical blocks, and the number of pieces required for each activity varies according to the suggested logical organization.

**Chart 1:** Description of materials: Activities 1, 2, and 3

| | ACTIVITY 1 | ACTIVITY 2 | ACTIVITY 3 |
|---|---|---|---|
| MATERIAL | Logic Blocks | Logic Blocks | Logic Blocks |
| DESCRIPTION | Twelve pieces selected from the 48 pieces of the material.<br>A base as a matrix to organize the pieces (rows x columns = 3 x 4). | Twenty-four pieces selected from the 48 pieces of the material.<br>A base made as a matrix to organize the pieces (rows x columns = 6 x 4). | Forty-eight pieces of the material (all pieces).<br>A base made as a matrix to organize the pieces (rows x columns = 6 x 8). |

**Source:** The authors.

To meet the learning characteristics of students with ASD identified in our study, we subdivided the complex tasks into sequenced stages, step by step. Each stage requires developing a new set of skills, which causes changes to the activity system in the object and the outcome to be achieved. From this perspective, internal contradictions arise in the activity system, as students are not prepared to deal with the new demand imposed by the change in the object.

The local historicity of the activity system shows that the change in the object, here represented by the mathematical object, is caused by the teacher after assessing the student's performance in an activity and the possibility of advancing to a new phase seeking, in this way, to promote the development of new skills driven by the contradictions provoked within the system.

The necessary changes that occur in activity systems and that encompass the development of new skills can be identified as expansive transformations of the activity because, as contradictions arise caused by the change in the object and, consequently, in the outcomes to be achieved, the activity system is adapted to meet new demands.

The entire journey –from changing the object to consolidating the activity system– is characterized as a complete cycle of expansive transformation. The complete cycle of changes that affect activity systems outcomes in the development of a more comprehensive number of skills and allows building skills for students with ASD.

Activities 1, 2, and 3 rely on the use of concrete material logical blocks. Their ultimate objective is to complete the mounting base with all objects that satisfy the logic of organizing the attributes of the parts.

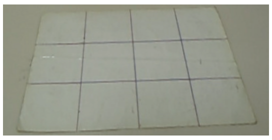
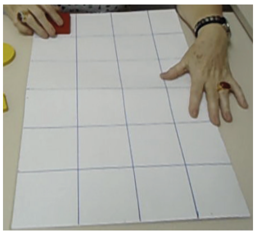
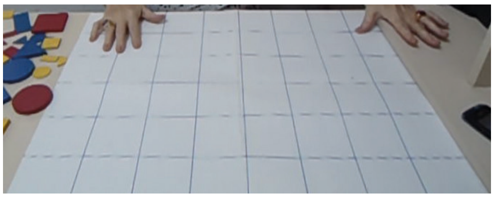
The mounting bases are made separately and follow the different matrices –or a single base with the largest matrix is made and, when used, folded to obtain the other configurations.

With the increase in the number of parts, the possible combinations of their attributes (shape, size, color, and thickness) increase, and consequently, more complex reasoning is required to compare, serialize, and classify objects.

Activities aimed at this audience must be planned to occur gradually and divided into phases because students with ASD may find it challenging to learn through complex tasks that require several steps or involve different concepts. Therefore, the tasks must be divided into stages to facilitate their understanding.

The logical organization of the pieces in the three activities will be determined according to the arrangement of the pieces in the first line and first column of this matrix: variation of shapes in the first line and color variation in the first column. In Figure 3, we show the mounting bases for each activity.

**Figure 3:** Mounting Bases (Activity 1, 2, and 3)

| ACTIVITY 1<br>(3 x 4 matrix) | ACTIVITY 2<br>(6 x 4 matrix) | ACTIVITY 3<br>(6 X 8 Matrix) |
|---|---|---|
|  |  |  |

**Source:** The authors.

The work environment must be organized before starting the activity. We recommend that teachers only keep objects that will be used during the activity; otherwise, the students' focus may be compromised. Takinaga (2015) states that some research indicates that students with ASD perform better in organized environments, which allow them to direct their focus of attention to what is taught, thus favoring learning.

Students with these characteristics can easily be distracted by the spinning of a fan or a noise imperceptible to others and tend to focus excessively on details, which can harm learning. Therefore, it is up to the teacher to ensure that the student's attention is focused on what is needed.

According to Gomes (2007, p. 346-347), "about responses to environmental stimuli, several researchers, in different theoretical orientations, have described limitations or changes in the way people with autism respond to stimuli." Furthermore, according to Lovaas et al.'s perceptions (1971 *apud* GOMES, 2007, p. 347),

[...] Children with autism generally learned to respond to part of a complex stimulus and did not focus on the stimulus. In other words, when the child was shown a complex figure with many details, they kept their attention on just one of the details and did not see the whole picture. The same reaction could also be observed in the simultaneous presentation of visual and auditory stimuli [...]. Generally, one element of the compound stimulus (visual or auditory stimulus) exerted discriminative control, while the other was apparently ignored.

Students with ASD may present behavioral problems when they must wait for something for long. Thus, the teacher must plan the activity so there is no waiting time for the start. It is also not recommended to carry out tasks that last too long. The duration of activities must also be considered in planning.

In this sense, regarding the proposed activities, the way the attributes of the pieces will be combined must be planned so that, when presenting the activities to the students, they can be started immediately.

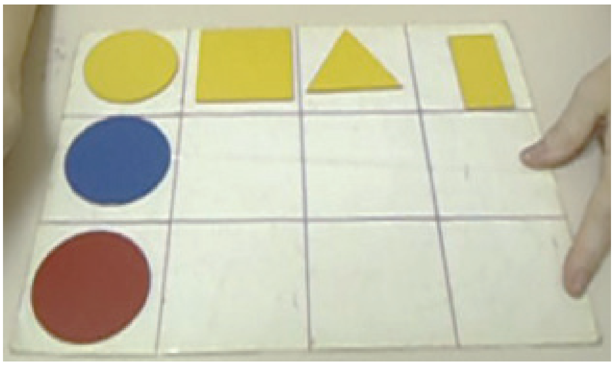
It is advisable to carry out activities for this purpose before starting work so that students become familiar with the attributes of the pieces. We suggest that the teacher delimit a space with a string, place a card signaling one of the attributes –for example, color– and paint the card with one of the desired colors.

The student must place all pieces identified with this attribute in that space, regardless of other characteristics. The objective is to work on one attribute at a time so that the student can identify and differentiate it from the others. This step may be unnecessary for typical students, as they will easily identify pieces when asked to compare more than one attribute –for example, selecting the yellow and large pieces.

After working with the recognition of the material and its attributes, Activity 1 will begin. One must use the mounting base with a 3 x 4 matrix for this activity. With the environment organized, the teacher shows the student the base with the first row and first column already filled in to respect the variation of colors in the column and shapes in the row. The other attributes must be kept for all other pieces, as the objective is to gradually develop the skills of comparison, classification, and seriation using the variation of just two attributes.

Figure 4 presents a suggestion for arranging the pieces for this first activity. We observed that all the pieces arranged on the base have two varying attributes –colors and shapes– but size and thickness remain the same, respectively, as large and thin.

**Figure 4:** Example arrangement of parts - Activity 1



**Source:** The authors.

After showing the partially assembled base to the student, the teacher points to a position in the matrix and asks them to insert the corresponding piece in that space: "Which piece comes here?" At this point, the student is expected to observe the logic of the organization of the parts already positioned on the mounting base.

If the student cannot select the correct piece, it may be necessary to help them and hand them the corresponding piece until they can identify it without assistance.

Students with ASD often have difficulty imitating other people's actions. For this reason, the instructions for all activity steps must be straightforward. If necessary, the mounting base must be completed with all the corresponding parts together with the student until they can perform the task independently.

During the execution of the activity, it is possible to work on the concepts of *intersection* when thinking simultaneously about pieces that meet more than one attribute, *comparison* when locating the corresponding part, and *seriation* when ordering the position of the parts on the mounting base.

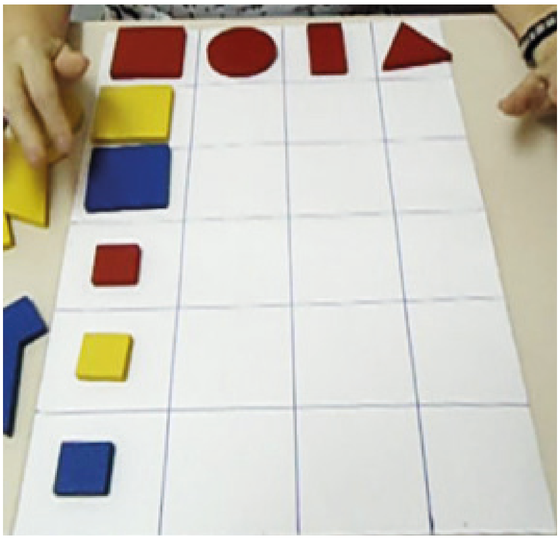
To help the student understand the concept of intersection, we suggest a movement with the hands: they start simultaneously from each object that presents the attributes considered and meet in the matrix position where the corresponding object will be placed. Figure 5 portrays this possibility.

**Figure 5:** Hand movement representing the concept of intersection

**Source:** The authors.

For Activity 2, the teacher will use the mounting base with a 6 x 4 matrix. As in the previous activity, the objective is to complete the base with the pieces corresponding to the intersection of the attributes previously inserted in the lines and columns of the mounting base. Figure 6 presents a suggested combination of parts and their attributes.

**Figure 6:** Example of arranging the pieces - Activity 2



**Source:** The authors.

After completely assembling the matrix from Activity 3, students must be assessed, and the teacher must explore different configurations and logical arrangements of the pieces. The student is asked to fill in the matrix sequentially and randomly.

Students with ASD often have difficulty generalizing their learning, so it is important to present different situations when working with the same material.

In the proposed activities, an error in the positioning of the pieces will prevent them from completing the matrix. Upon realizing the error, a typical student will try to correct it.

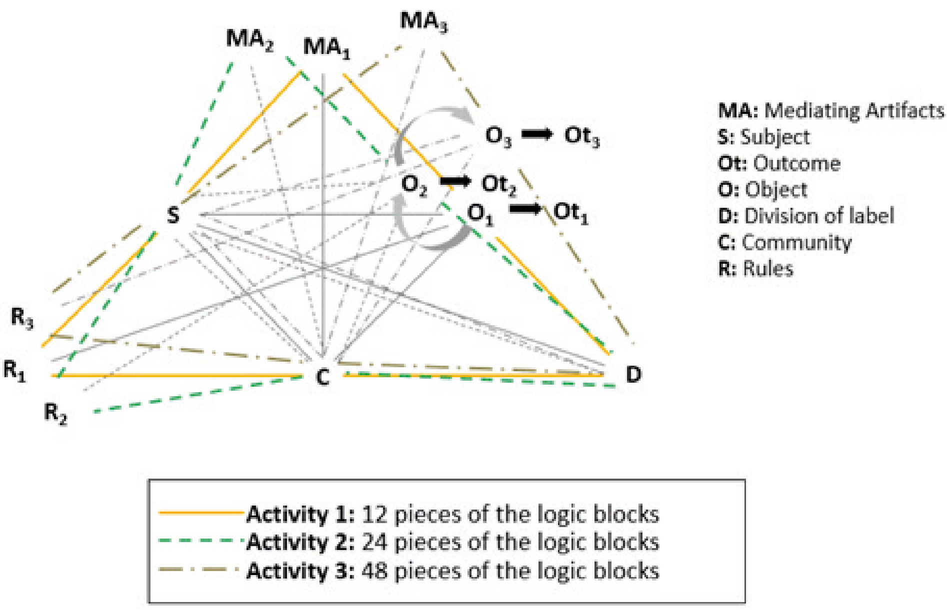
Students with ASD, however, have difficulty learning through error. Learning must, therefore, occur error-free. *Error-free learning* consists of preventing errors from occurring during the teaching process of these students. In this sense, an error in the positioning of a part must be corrected immediately.

Signaling the end of an activity is essential because students with ASD may become anxious during the transition time between tasks. Expressions such as “It’s over! Well done!” signal the end of an activity and mean predictability, which can reduce those behaviors.

We conclude those analyses of activities through their representation in an activity system and its elements, the basic unit of analysis of the activity theory, and named based on the skills developed to *classify*, serialize, and *establish correspondence*.

Activities 1, 2, and 3 are represented in the activity system shown in Figure 7. The elements for those activities are determined as follows: Logical Blocks (MA); Student with ASD (S); Development of skills to classify, serialize, and establish correspondence (Ot); Mounting bases (O); Teacher as mediator and student as apprentice (D); Teacher and students (C); and Guidelines for positioning parts in the first row and first column of the mounting base and learning characteristics of students with ASD (R).

**Figure 7:** Activity system: classify, serialize, and establish correspondence




**Source:** The authors.

For this activity system, we highlight the following learning characteristics of students with ASD: structured teaching, use of concrete material, visual guidelines for carrying out activities, organized environments with few stimuli, predictability, and favoring generalizations.

The diagram in Figure 7 shows the elements that make up the “Activity system: classify, serialize, and establish correspondence” and their relationships, which allows us to identify in Activities 1, 2, and 3 contributions that can favor teaching planning so that the development of skills that precede the mathematics learning of students with ASD occurs.

Each larger triangle in Figure 7 is associated with an activity. The upper part represents Vygotsky's (1978) mediation model, in which the relationship between Subject (S), Object (O), and Mediating Artifacts (MA) is established. At the bottom are the constituent components of collective action that can influence these relationships: the Rules (R), the Community (C), and the Division of Labor (D).

Arrows that direct movement between objects $O_1$, $O_2$, and $O_3$ indicate the changes of objects –here represented by mounting bases– as they allow the variation of the attributes of the parts to be considered for their positioning in the assembly matrix, to encompass broader possibilities and favorable to the development, by students with ASD, of skills to compare, serialize, and establish correspondence.

In this article, we do not propose to analyze the relationships between the elements that compose the "Activity system: classify, serialize, and establish correspondence" presented in Figure 7. Finally, we will stress which elements one must consider when planning teaching activities to develop skills favorable to teaching mathematical objects to students with ASD.

The mediating role of Mediating Artifacts in the development of the abilities to classify, serialize, and establish correspondence occurs through the manipulation of this artifact –the concrete material Logical Blocks– designed to develop such abilities.

Indirectly, structuring the environment in which there is a need for collective coexistence can somehow favor the satisfactory performance of people with ASD.

Among the rules that govern the activity system, we considered the listed learning characteristics of students with ASD, because they contribute to the teaching and learning process. We emphasize that such characteristics must be observed individually because, as autism is classified as a spectrum, several variations must be considered.

We move on to our final considerations, in which we seek to emphasize the aspects highlighted in the proposed activities that can contribute to the planning and organization of teaching with a view to the learning of the object "numbers and construction of their meanings" that precedes its representation and the operations for students with ASD.

## 4. Final considerations

We base our final considerations on our objective, which was to present part of the analysis of the data collected in the scope of a research work that sought answers to the question: "What elements can contribute to the process of mathematics teaching to students with autism spectrum disorder (ASD)?" (TAKINAGA, 2015, p. 29).

We concluded that, for students with ASD to learn mathematics effectively, the teaching process must consider elements that allow the planning of teaching activities based on the construction of meanings of the mathematical objects studied.

The theoretical framework adopted provided the necessary lens to identify the relationships between the elements that underpin the planning and execution of activities designed to enable accessible mathematics teaching to students with ASD.

The activities presented in this article were intended to develop the skills of classifying, serializing, and establishing correspondence. Such skills precede the teaching of mathematical objects such as "numbers and their operations" and "algebra."

We highlight the following aspects as initial possibilities before planning the teaching of the object "numbers" for students with ASD: adequate choice of materials that allow the construction of meanings; use of different materials to develop a skill or a set of skills to provide diverse teaching situations so that the student can generalize learning; structuring and systematizing teaching activities in stages so that learning occurs gradually; maintaining an environment free of stimuli that could divert students' focus of attention –for example, colors and noises, among others–; and consideration of the learning characteristics of this audience and their individualities.

Among the planning proposals presented, Takinaga (2015, p. 113) states that "only the structuring of teaching, its systematization into steps, does not guarantee the mathematical learning of students with ASD; it is necessary to construct meanings, their properties, of the mathematical objects involved."

Based on the activities presented, the choice of concrete materials contributed to constructing the meanings of the numerical skills to be developed. However, this resource should be considered the initial support and replaced gradually to favor the construction of abstract thinking and effective work with mathematical objects and their representations.

One must be attentive to the design of activities that gradually reduce students' dependence on concrete material and enable the development of their abstract reasoning. We know that there is a wide range of individuals on the autistic spectrum who have different levels of cognitive development; however, even given this finding, we can say nothing about their ability to develop abstract thinking (TAKINAGA, 2015).

We conclude by highlighting the importance of considering the implications of teaching planning and its influence on providing learning that enables learning for everyone, including students with ASD. The choice of strategies must encompass essential elements for constructing the targeted mathematical knowledge and developing skills to make learning meaningful and accessible for this audience.

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**Editorial History**
Recebido em 12/12/2023.
Aceito em 20/03/2024.
Publicado em 27/04/2024.

**How to cite – ABNT**
TAKINAGA, Sofia Seixas; MANRIQUE, Ana Lúcia. Possibilities in teaching planning to make mathematical knowledge accessible to students with autism spectrum disorder. **REVEMOP**, Ouro Petro/MG, Brasil, v. 6, e202402, 2024. https://doi.org/10.33532/revemop.e202402

**How to cite – APA**
TAKINAGA, S. S., & MANRIQUE, A. L. (2024). Possibilities in teaching planning to make mathematical knowledge accessible to students with autism spectrum disorder. *REVEMOP*, *6*, e202401. https://doi.org/10.33532/revemop.e202402

This article was funded by the Minas Gerais State Research Support Foundation (Fundação de Amparo à Pesquisa do Estado de Minas Gerais - FAPEMIG), Project APQ-04960-23, Notice N. 008/2023 - Support program for scientific and technological publications.