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Problem Solving from The Man Who Calculated

Resolução de problemas a partir da obra O Homem que Calculava

Resolución de problemas a partir de El Hombre que Calculaba

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Abstract

This article describes the analyses of the mathematical knowledge mobilized in the mathematics teaching-learning process through problem solving based on the book *The Man Who Counted*, authored by Malba Tahan. This inquiry-based qualitative research is supported by the theoretical assumptions of problem solving as a teaching methodology. The investigation was developed in a public school in Benevides (PA), with students of the 8th grade of elementary school, during the Supervised Practicum. The analyses were carried out from the registers of the resolution of two problems taken from Malba Tahan's book, the case of the 21 casks and the four fours. The research pointed out in its results that working with problem solving requires a new attitude from the teacher and the students. It made it possible to reflect on problem solving as a tool for mathematical investigation.
Keywords: Problem Solving. Mathematics Education. Malba Tahan.

Resumo

Este artigo tem como objetivo analisar os conhecimentos matemáticos mobilizados no processo de ensino-aprendizagem da matemática por meio da resolução de problemas a partir do livro *O Homem que Calculava*, de autoria de Malba Tahan. A pesquisa está fundamentada nos pressupostos teóricos da resolução de problemas como uma metodologia de ensino. A investigação foi desenvolvida em uma escola pública de Benevides (PA), com alunos do 8º ano do ensino fundamental, durante o Estágio Supervisionado. Uma pesquisa qualitativa do tipo exploratória. As análises foram realizadas a partir dos registros da resolução de dois problemas retirados livro de Malba Tahan, o caso dos 21 vasos e os quatro quatros. A pesquisa apontou em seus resultados que o trabalho com a resolução de problemas requer uma nova postura do professor e dos alunos. Possibilitou refletir sobre a resolução de problemas como uma ferramenta de investigação matemática.
Palavras-chave: Resolução de Problemas. Educação Matemática. Malba Tahan.

Resumen

Este artículo tiene como objetivo analizar el conocimiento matemático movilizado en el proceso de enseñanza-aprendizaje de las matemáticas a través de la resolución de problemas a partir del libro *El Hombre que Calculaba*, de autoría de Malba Tahan. La investigación se basa en los supuestos teóricos de la resolución de problemas como metodología de enseñanza. La investigación se desarrolló en una escuela pública de Benevides (PA), con alumnos del 8º año de la escuela básica, durante la Pasantía Supervisada. Una investigación cualitativa exploratória. Los análisis se realizaron a partir de los registros de la resolución de dos problemas tomados del libro de Malba Tahan, el caso de los 21 jarrones y los cuatro cuatros. La investigación señaló en sus resultados que trabajar con la resolución de problemas requiere una nueva actitud por parte del profesor y de los estudiantes. Permitió reflexionar sobre la resolución de problemas como herramienta de investigación matemática.
Palabras clave: Resolución de Problemas. Educación Matemática. Malba Tahan.

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1. Introduction

The importance of problem solving in teaching and learning has grown in recent decades due to national and international discussions about the basic education mathematics curriculum. Mathematics educators, such as George Polya in the United States in the 1960s, began to reflect on mathematics teaching from the perspective of memorization and repetition of algorithms, concluding that pedagogical didactic work needed improvement. In this way, classes could not continue assuming a trend that considered the construction of knowledge through routines of algorithmic application procedures; on the contrary, students had to be active in this process.

Under this conception, mathematics would be seen as a "teaching object" in which the mathematician discovers mathematics in an outer reality. However, after discovering a mathematical result, they must justify it within a formal structure, and only then is it ready to be taught. In other words, mathematics teaching should be based on understanding the mathematical object and its relationship with the sociocultural environment.

In this context, the generating question that guided this investigation was: What characteristics of activities in the problem-solving context can facilitate participants' mobilization of mathematical knowledge? Our research proposal used Polya (1986), Onuchic (1999), Onuchic and Allevato (2004), and Pozo and Echeverria (2002), among other authors who study problem solving in the mathematics teaching-learning process to answer this question.

From this perspective, this research –situated in the context of the research project entitled *Temas Emergentes na Formação de Professores que Ensinam Matemática* [Emerging Themes in the Education of Teachers who Teach Mathematics], linked to the Postgraduate Program in Science and Mathematics Education at the Federal University of Pará (PPGECM-UFPA)– *analyzes the mathematical knowledge mobilized in the teaching and learning processes of mathematics through problem solving based on Malba Tahan's book "The Man Who Counted*." Regarding methodological aspects, this study took a qualitative, inquiry-based approach. The research was carried out in an 8th-grade elementary school class in a public school in Benevides (PA) during the Supervised Practicum subject[3].

The research was developed during the Supervised Practicum. The didactic-pedagogical activities were organized based on two problems taken from Malba Tahan's book *The Man Who Counted*. They were as follows: the problem of "the 21 casks" and the problem of "the four fours." The research was carried out over two weeks of participatory observation of mathematics classes, accounting for twelve hours of activities at the school. The empirical material results from students' registers, which we analyzed to reflect on the mathematical knowledge mobilized during classes, especially problem-solving strategies.

Generally, the research showed, regarding the mathematical learning process, that through problem solving, students could express their ideas about mathematical knowledge and build, in partnership with the teacher and colleagues, skills to solve problems more independently. Further-

3 The Supervised Practicum is a mandatory subject for the mathematics teaching degree offered by the Mathematics College of the Castanhal University Campus, of the Federal University of Pará.

more, the proposal presented mathematics as a research activity, as they had to carefully understand and interpret the situations posed in each problem.

Below, the text is organized into four sections. The first section discusses problem solving as a mathematics teaching methodology. The second section presents the epistemological aspects of the research methodology and a description of how it was developed. Then, the third section presents the research results and discusses the knowledge that emerged from the investigative process. The last part of this text concludes with final reflections on the scientific and practical knowledge mobilized in this research.

2. Perspectives on Problem Solving in Mathematics Classes

Polya (1986, p. 15) states: “A great discovery solves a great problem, but there is always a pinch of discovery in solving any problem.” In this sense, mathematics teaching faced old dilemmas regarding the teacher-student-learning process, which stemmed from the inefficiency of communicating conceptual content, adding to the student’s difficulty in understanding and consolidating the content. These obstacles in the mathematics teaching-learning process, currently combined with the need for interconnected monitoring of teaching with technology, as well as the biases that accompany them (lack of teaching-pedagogical materials, structural resources, planning time), thus generating a need for adaptation and innovation that would help teachers seek more effective ways to teach.

Thus, studies (POLYA, 1986; ONUCHIC, 1999) on problem solving as a methodology for teaching mathematics have gained space in academic research, aiming to improve students’ cognitive skills and active participation in the teaching-learning process. In this research, problem solving as a teaching methodology, mainly in basic education, has sought to provide students with the possibility of seeing mathematics as a dynamic subject and a way of reading the world through mathematical language.

In these terms, according to Polya (1986), the effective search for teaching how to solve problems consists not only of providing students with dexterity and consistent strategies but of stimulating the habit and action of receiving learning as a problem in which one must find answers. However, we cannot confuse a mere exercise with a problem situation in classroom practice. Students must be clear about the distinction between mathematical exercises and mathematical problems. They need to understand their role in the proposed tasks instead of repeating algorithms demonstrated by the mathematics teacher.

When adopting problem solving as a teaching methodology, the teacher must give students autonomy instead of encouraging heteronomy. Regarding this, Polya (1986, p. 04) states that:

> Students must acquire as much experience as possible by working independently. Nevertheless, if they are left alone, without help, or with insufficient help, they may not experience any progress.

One of the biggest challenges of teaching is finding a methodology that applies to the majority of students, seeking to direct them not only to the correct result but also to see the different applications that each problem has and its relationship with everyday life, which is not easy, as it requires time, experience, dedication, and consistent concepts. Polya (1986) affirms that the teacher must find the right measure of assistance for the student so there is no shortage or excess, allowing the student to construct knowledge individually.

According to Onuchic (1999), in the 1980s, problem solving gained prominence in research on mathematics teaching and learning. These discussions focused on classroom work, teaching strategies, suggestions for types of problems, and learning assessment. Part of these materials contributed to mathematics teachers' practice. In this same context, guiding documents for the construction of a new curriculum for teaching mathematics emerged in the United States of America (USA).

These curriculum guidelines were not intended to "tell, step by step, how to work with these documents. On the contrary, they wanted to present objectives and principles in defense of which curricular, teaching, and assessment practices could be examined" (ONUCHIC, 1999, p. 10). This means that the curriculum guidelines sought to expand discussions on educational policies, teacher education, family participation in students' school lives, and the organization of mathematics programs for all basic education.

Onuchic (1999, p. 11) says:

> [...] Another characteristic of these curricula is the use of contexts in problem solving to develop mathematical content and make connections with other areas. These curricula portray mathematics as a subject unified by coherently integrated topics.

These international discussions on a new curriculum proposal for mathematics teaching impacted the Brazilian education context. In 1997, the Ministry of Education (Ministério da Educação–MEC), through the Basic Education Office (Secretaria de Educação Básica–SEB), after several studies, published the National Curriculum Parameters (Parâmetros Curriculares Nacionais–PCN). Thus, in 1998, the Mathematics PCNs for the final years of elementary school was published. This document includes discussions on cross-cutting themes (ethics, health, environment, citizenship, sexual orientation, cultural plurality, work, and consumption) and reflections on problem-solving and the mathematics teaching-learning process (BRASIL, 1998).

Problem solving in the PCNs, based on research in the mathematics education field, was identified as the starting point for mathematics classes. In other words, mathematics teaching had to overcome the mere reproduction of algorithms and the accumulation of information conveyed by the teacher. From the perspective highlighted in the document, mathematical knowledge would gain meaning "when students faced challenging situations to solve and worked to develop resolution strategies" (BRASIL, 1998, p. 40). The PCNs criticized the application-centered mathematics teaching students acquired previously. Problem solving could be worked on in the knowledge production process and not as a way of applying what had already been taught.

Mathematics teaching was being discussed in an educational context in which didactic-pedagogical practices consisted of "teaching a concept, procedure or technique and then presenting a problem to assess whether students are capable of using what was taught to them" (BRASIL, 1998, p. 40). In this way, the idea that solving a mathematical problem was applying an already learned procedure was reinforced, i.e., just calculating, based on the numerical information contained in the problem statement. This practice implied focusing work on results and not on the process.

Mathematical knowledge seen from this perspective was presented to students as ready and finished rather than a set of concepts and procedures interrelated and integrated with other areas of knowledge. In these terms, it was assumed that learning occurred through the reproduction or application of ready-made techniques. Therefore, the problem solving discussed in the PCNs,

> [...] allows students to mobilize knowledge and develop the ability to manage the information at their fingertips. Thus, students will have the opportunity to expand their knowledge about mathematical concepts and procedures as well as broaden their view of problems (BRASIL, 1998, p. 40).

From this perspective, mathematics teaching took another path, as problem solving became the guiding axis of the mathematics teaching and learning processes. The definition ceased to be the starting point of the class, and the problem situation or the generating problem became the triggering activity. Taking a generating problem, the teacher could explore mathematical concepts, ideas, and procedures more closely. In other words, the class uses problem situations to help the students develop and expand problem-solving (heuristic) strategies, which also implies the development of autonomy and self-confidence.

In the PCNs, the conception of a mathematical problem is extended beyond a mere exercise, a mere application of a formula or operational process. According to the parameters, "there is a problem only if students are led to interpret the statement of the question posed to them and to structure the situation presented to them" (BRASIL, 1998, p. 41). In this way, mathematical learning is articulated with other areas of knowledge, just as the student understands mathematical concepts through articulation with other already learned concepts.

Problem solving can provide contexts for learning conceptual, procedural, and attitudinal content. Thus, a mathematical problem is "a situation that demands the performance of a sequence of actions or operations to obtain a result" (BRASIL, 1998, p. 41). This shows that the solution to a problem is built during the resolution process and that a mathematical problem can be relative; that is, what may be a problem for one student is not a problem for another. Therefore, what determines a problem situation is the challenge proposed.

Regarding the process of solving a mathematical problem, for Polya (1986), the first step to finding the solution is to understand the problem, i.e., to understand what the unknown and the data are. The next step is to establish a plan. Here, the student needs, for example, to resort to similar problems. The third step is to execute the plan; then, it is time to put into practice what was planned to find a solution to the problem. Finally, take a look back. At this stage, the student must check whether the solution satisfies the problem, meaning that they must test the solution.

For Onuchic (1999), the steps for solving a problem and the heuristics for seeking the solution, as Polya describes and discusses, have been adopted in curriculum guidelines, regardless of the mathematical content. These discussions encouraged problem solving as a teaching methodology. From this perspective, "the student either learns mathematics by solving problems or learns mathematics to solve problems" (ONUCHIC, 1999, p. 211), which means that teaching mathematics through problem solving is no longer an isolated process.

For Onuchic and Allevato (2004, p. 5),

> Research on mathematical problem solving has received much attention in recent decades. Notable developments include George Polya's pioneering work on problem solving. Furthermore, other studies have been developed: teaching problem-solving strategies, heuristics, metacognitive processes, and mathematical modeling.

From this research, mathematics teaching began to overcome the didactic-pedagogical work centered on repeating and memorizing basic arithmetic facts, such as the excessive use of multiplication tables. In this context, the theoretical-methodological proposal was to avoid that mathematics teachers spent most of the class time speaking. At the same time, the students received information, wrote, solved examples, and then solved infinite lists of exercises. According to Onuchic (1999, p. 201), students repeated "exercises done in the classroom and trained at home. The student's knowledge received through repetition was measured through tests." Thus, the exercises were solved the same way the teacher had done, always following the same resolution steps.

In repetition-centered classes, some students could understand what they were doing; that is, they could "think" about the mathematics studied. However, most students did not understand the mathematical algorithms studied in class, as "most forgot what they had memorized in a short time" (ONUCHIC, 1999, p. 202). In this context, the mathematics curriculum in Brazilian schools was not yet well defined, but the classes focused on arithmetic, algebra, and geometry.

Over the years and with the advancement of research in mathematics education (POLYA, 1986; ONUCHIC, 1999; ONUCHIC & ALLEVATO, 2004), the teaching and learning process sought to reflect on the importance of students learning mathematics with understanding. Thus, problem solving, especially in academic research, has assumed three general characteristics: as a mathematical skill, as art, and as context (ONUCHIC & ALLEVATO, 2004). Despite the growing interest in the topic, the studies contributed little to classroom practice, given the complexity of the proposals.

The little impact of academic research in the classroom was also because problem solving appeared as an isolated topic, focusing on understanding concepts and developing algorithms followed by applying problems with statements; for example, a combinatorics problem would ask us to draw an anagram. Regarding this, Pozo and Echeverria (2002, p. 6) highlight that:

> [...] Despite these decades of research and associated curriculum development, it appears that students' problem-solving skills still require substantial improvement, especially given the rapidly changing nature of today's world.

Therefore, it is necessary to reflect on the importance of new perspectives on developing research that addresses problem solving as an object of investigation. Consider building a theory that articulates problem solving with teaching and learning activities in basic education. In these terms, research and teaching in problem solving should contribute to students expanding and deepening their heuristic strategies in that approach, as well as "teaching metacognitive strategies, developing ways to improve students' beliefs about the nature of mathematics and their skills in mathematical knowledge" (POZO & ECHEVERRIA, 2002, p. 8).

In addition to skills specific to mathematics, working with problem solving can expand and consolidate knowledge related to reading and writing. In these terms, this research considers a mathematical problem as a textual genre. Therefore, there is a need to teach and promote learning situations in order to read these texts because some words may have different meanings (of a mathematical nature), making understanding difficult. According to Itacarambi (2010, p. 14), "questioning the interpretation of the text, most of the time, helps evaluate the answers given by students and to verify that the teacher's interpretation is not the only possible one." Thus, the mathematical problem becomes a contextualization instrument, from the moment it proposes situations that require a mathematical solution and lead to questioning, research, and the insertion of operations within a context (ITACARAMBI, 2010).

Reading in mathematics classes through problem solving can be considered a teaching practice. According to Fonseca and Cardoso (2009, p. 66):

> Reading texts that have as their object mathematical concepts and procedures, the history of mathematics, or reflections on mathematics, its problems, methods, and challenges can, however, do much more than guide the execution of a certain technique, adding elements that not only favor the constitution of meanings of mathematical content but also contribute to the production of meanings of mathematics itself and its learning by the student.

The mathematics teacher can guide, practice, or facilitate readings of mathematical texts in partnership with the Portuguese language teacher, not only from the perspective of teaching mathematics but also from the perspective of developing reading comprehension. Teachers can select some texts provided to students that contain numerical information; information conveyed through graphs and tables, map reading, texts with mathematical ideas, etc.

A necessary practice in mathematics classes is writing. Writing does not have the same fluidity as speaking because, when we write, we cannot easily resort to the various arguments available in oral interaction. However, Smole and Diniz (2001, p. 23) state that "writing joins oral and drawing to be used as another resource for representing students' ideas." Therefore, writing and drawing expressing mathematical ideas must be encouraged in classes, especially in problem-solving activities.

However, expressing oneself through mathematical language is not simple, as its language requires rigor. By demanding from students a language that we consider appropriate and precise, we run the risk of preventing some of them from having access to the meaning of mathematical statements constructed from an approximate language in work where the important thing is to articulate meanings, relate ideas, and stage of reasoning.

Writing and registering through drawings help mathematical learning significantly, as they encourage students to deal with problem situations, reflect on mathematical operations, clarify mathematical ideas and concepts, and are a catalyst for discussions in group activities, above all, help students to learn what is being studied (SMOLE & DINIZ, 2001).

Mathematics teachers can explore working with problem solving since, in this methodological perspective, students can resort to various mathematical concepts and ideas. Thus, problem solving is seen in this research proposal as an investigation activity whose starting point is qualitative analysis, i.e., having an idea of the situation, delimiting it, and having clear objectives, that is, what one seeks. Therefore, having a clear concept of the problem is the first step for the teacher to understand the students' production.

Therefore, mathematics problem-solving classes can be planned with Portuguese language and science classes. To achieve this, when reading and writing problem situations, teachers must have the will and creativity to reorganize their classes. They must move away from rigid propositions (definition-example-exercise) towards more meaningful and contextualized didactic ones.

The following section will present how we worked with problem solving in mathematics classes.

3. Research Methodology

This research, which aimed to analyze the mathematical knowledge mobilized in the mathematics teaching and learning process through problem solving, took as a reference the book *The Man Who Counted*, by Malba Tahan, through a qualitative and inquiry-based approach (FIORENTINI & LORENZATO, 2006). Therefore, this section presents the context of the investigation, the problems worked on in mathematics classes, the instruments used to construct the information, and the method of analyzing the research results.

The research context was an 8th-grade elementary school class in a public school in Benevides (PA). The interest arose during the development of the Supervised Practicum, a curriculum component of the mathematics teaching degree course at the Federal University of Pará. On this occasion, observation, participation, and activity-conducting were carried out under the supervision of the class mathematics teacher. The practicum workload was 120 hours, and part was allocated to this research.

The Supervised Practicum was an opportunity to discuss and reflect on aspects related to teaching, i.e., a defining moment of the teaching degree course. Therefore, there was an interest in carrying out research anchored in the teaching and learning process, as the course qualified mathematics teachers, and researching the context of professional practice would be relevant to the researcher's professional development. The choice of the school arose due to the researchers having access to the work carried out by the institution and the interest of the management team and the class teacher in participating in this research. Therefore, this rapprochement with the management team and the class teacher facilitated insertion into the locus of research.

After this approach, during the math teacher's planning, the book *The Man Who Counted*, by Malba Tahan (2010), was introduced. The teacher was interested in knowing what the story was about. We explained that the author wrote an adventure narrative and that these stories had much mathematics. After this conversation, the teacher became interested in developing some classes using this book. Then, the teacher provided two weeks of classes for the research to be carried out. Therefore, during twelve hours of classroom activities, students performed reading and writing tasks using problem solving, a methodology adopted by the teacher.

The research process considered mathematics content from the 8th grade of elementary school. Based on the teacher's planning, we identified the following subjects: "numbers and algebraic thinking." Therefore, we read the book to select episodes related to the mathematics content of the planning. Thus, two episodes were chosen: “The Case of the 21 Casks” and “The Case of the Four Fours”. They were taken from the book *The Man Who Counted*, by Malba Tahan (2010), the pen name of the Brazilian mathematics teacher and writer Júlio César de Mello e Souza. Therefore, it is a narrative-type work in the novel genre (for children and young people) that addresses mathematical knowledge and Muslim culture.

The mathematical knowledge covered in this work is related to fundamental operations (addition, subtraction, multiplication, division), potentiation, radiciation, fraction operations, algebraic thinking, equations, and intuitive series. In this book, the main character, Beremiz Samir (the counting man), is interested in mathematics and is renowned for solving problems. His skills raise curiosity, openness, and envy in others wherever he goes.

The first problem worked on with the students was the episode “The Case of the 21 Casks.”

> The eloquent man who counted was about to continue his strange observations about the sacred number, when we saw, at the door of the inn, our dedicated friend Sheikh Salem Nasair, who waved repeatedly, calling for us.
> Sheikh Salem said: – I'm happy to have found you now, O counting man! Your arrival, not only for me, but also for three friends who are staying at this inn, was highly providential.
> And he added with friendliness and visible interest:
> - Come! Come with me, this is a very serious case.
> He took us inside the inn. He led us through a dark, damp corridor to the inner courtyard, which was welcoming and bright. There were five or six round tables there. Next to one of these tables were three travelers who seemed strange to me.
> When the Sheikh and the calculating man approached, they got up and saluted them. One of them looked very young. He was tall and thin, had clear eyes, and wore a beautiful egg-colored yellow turban with a white bar where an emerald of rare beauty sparkled. The other two were short, broad-shouldered, and dark-skinned like African Bedouins.
> The Sheikh said, pointing to the three Muslims:
> - Here are, O counting man, the three friends. They are sheep farmers in Damascus. They now face the most curious problems I have seen. And this problem is the following: As payment for a small batch of sheep, they received here, in Baghdad, a very fine batch of wine made up of 21 equal casks, 7 of which were full, 7 half full, and 7 empty.
> They now want to divide the 21 casks so that each receives the same number of casks and the same portion of wine. Dividing the pots is easy. Each partner must have seven casks. In my opinion, the difficulty is sharing the wine without opening the casks, that is, keeping them exactly as they are. Is it possible, O calculating man, to obtain a solution to this problem?
> After meditating in silence for two or three minutes, Beremiz replied:
> - O Sheik, the division of the 21 casks you have just presented can be done without major calculations. I will indicate the solution that seems simplest to me (MALBA TAHAN, 2010, p. 53-54).

This first problem raised students' curiosity and interest. The teacher felt the difference in student participation in class. This problem was seen in six class hours in the first week of class. The problem “The Four Fours” was used in the second week of class.

> [...] Beremiz was interested in an elegant and harmonious light blue turban that a Syrian, somewhat hunchbacked, offered for 4 dinars. This merchant's tent was, in fact, very original, as everything there (turbans, boxes, daggers, bracelets, etc.) was sold for 4 dinars. There was a sign, in bright letters, that said:
>
> **“THE FOUR FOURS”**
>
> When I saw Beremiz interested in purchasing the blue turban, I objected:–I think it's crazy to buy this luxury. We are low on money and haven't paid for the inn yet.–It's not the turban that

> interests me – Beremiz retorted. – Notice that this merchant's tent is titled "The Four Fours." There is an astonishing coincidence in all of this, worthy of attention.–Coincidence? Why?
>
> - Now, Bagdali –Beremiz retorted– the caption in this table recalls one of the wonders of calculus: we can form any number using four fours! And before I questioned him about that enigma, Beremiz explained, scratching in the fine sand that covered the floor:–Do you want to form a zero? Nothing could be simpler. Just write:
>
> 44 – 44
>
> - There are four fours forming an expression that is equal to zero. Let's move on to number 1. Here is the most comfortable way:
>
> 44 : 44
>
> - It represents this fraction, the quotient of dividing 44 by 44. And this quotient is 1. Do you want to see number 2 now? (MALBA TAHAN, 2010, p. 28-30)

With these texts in hand, the teacher, in partnership with the researchers, planned the classes. In the first moment of the class, we explained how the work with the book would be carried out. Thus, to present the problem, we explained that the elements had been taken from Malba Tahan's book, *The Man Who Counted*, a narrative about Beremiz Samir's adventures. We also explained that the real author of the book is Júlio César de Mello e Souza, a teacher and mathematician who took who wrote under the pen name of Malba Tahan. The work captured the attention of the entire class because through the adventures and romantic atmosphere, the students enjoyed reading and began to look at mathematics from a more meaningful perspective.

The book was then made available in digital format to all students in the class, which helped organize the activities. Therefore, the first classes were investigative. In the following activities, printed copies of one of the problems from Malba Tahan's collection were distributed, and individual reading was carried out. On the same day, after individual reading, the class was organized into small groups to read collectively and discuss the story. After reading in groups, the next task was to read and interpret the proposed text, contributing to understanding the problem. The next task was solving the problem "The 21 Casks." Students would have to share what was told in the story. This problem-solving activity took two classes, as students initially had difficulty carrying it out. However, ideas emerged in the interaction with colleagues and the teacher's mediation.

The groups' solutions were recorded in the notebook and discussed so they could register them on the whiteboard, as these were instruments for communicating mathematical ideas. With the activities recorded on the whiteboard, discussions began about the several solutions that, a priori, satisfied the problem—a moment of interaction and exchange of ideas. To close the class that dealt with the first problem, the teacher formally presented the content and mathematical ideas emerging from the activity. This work script was used in both problems.

We used adequate tools to construct the information necessary for the research, participant observation, i.e., "a structured observation based on previous planning regarding data collection" (MINAYO, 2015, p. 79); then, we made a preliminary visit to the school and the class and talked with the mathematics teacher about organizing the activities and the materials the students would produce. The participant observation technique enabled the construction of a field diary –"a notebook in which we wrote all the information that is not part of the formal research material" (MINAYO, 2015, p. 71)– where observations were written. Therefore, students' writing and drawings were analyzed to reflect on the knowledge mobilized during classes and resolution strategies.

The students' activity sheets were the main empirical material for the analyses. The registers of the algorithms used to face problem situations also served as information for reflections on the resolution strategies and mathematical knowledge mobilized in the activities. These written records produced by the students were analyzed to reflect on the mathematical knowledge mobilized during classes, especially the strategies (heuristics) for solving problems. This material served as support for discussions of the results. With this experience in hand, we thought of an analysis method that was articulated with the information construction technique and the material produced during the research. Therefore, we used Minayo's *interpretative analysis methodology*. This method articulates the following actions: description, understanding, and interpretation. These actions must be linked with the theory that supported the discussion of the research object (MINAYO, 2015).

Therefore, this method of analysis seeks to make sense of what was described and analyzed. It is an analytical process that facilitates the interpretation of speeches, texts, drawings, documents, and books through a systemic process. This method analyzed the information constructed in/from the problem-solving process, as presented in the next section.

4. The Man Who Counted

This research considers that working with problem solving seeks to recover its productive role. Thus, the research enabled reflections on a student who needs to be encouraged to question, solve problems, create resolution strategies, and build mathematical knowledge independently, with the teacher as a mediator in this process. In this sense, this research assumed two lines of interpretative analysis: one of them emphasized the reading and writing activities of mathematical language and its interface with the mother tongue based on the problem "The 21 Casks"; the other discussed mathematical knowledge and problem-solving (heuristic) strategies "The Four Fours."

The research showed that writing and reading through problem solving could bring students closer to learning the mathematical language while learning their mother tongue and deepen mathematical ideas. An example of this can be seen in the students' productions in the episode "The 21 Casks," taken from Malba Tahan's book.

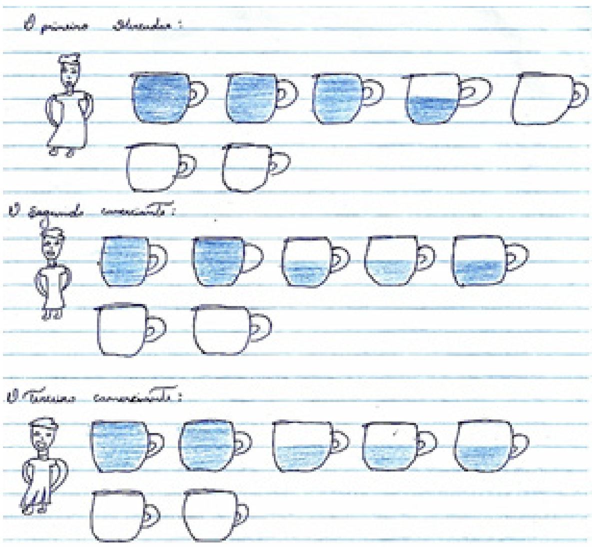
Figure 1: The Sharing of the 21 Casks

Source: Field Diary, 2023.

Figure 1 highlights the importance of registering with drawings. The student expressed their way of facing the problem situation. These characteristics make it possible to consider the importance of learning mathematics to be able to read texts and reading texts to learn mathematics. This practice must be more present in basic education mathematics classes as, according to Onuchic (1999, p. 201), "students should learn mathematics with understanding." This work allows the student to become familiar with the language and symbols specific to mathematics.

According to Smole and Diniz (2001), teachers must organize reading routines, regardless of Portuguese language classes, that articulate moments of individual, oral, silent, or shared reading so that, in mathematics classes, students face effective situations and diversified reading. Reading books, newspapers, maps, electricity bills, game rules, etc., are necessary practices; however, it is challenging to train proficient readers, as it involves several cognitive, affective, and social processes. Figure 2 illustrates the strategies used to resolve the problem situation in this context.

Figure 2: Representation of the Sharing of Casks



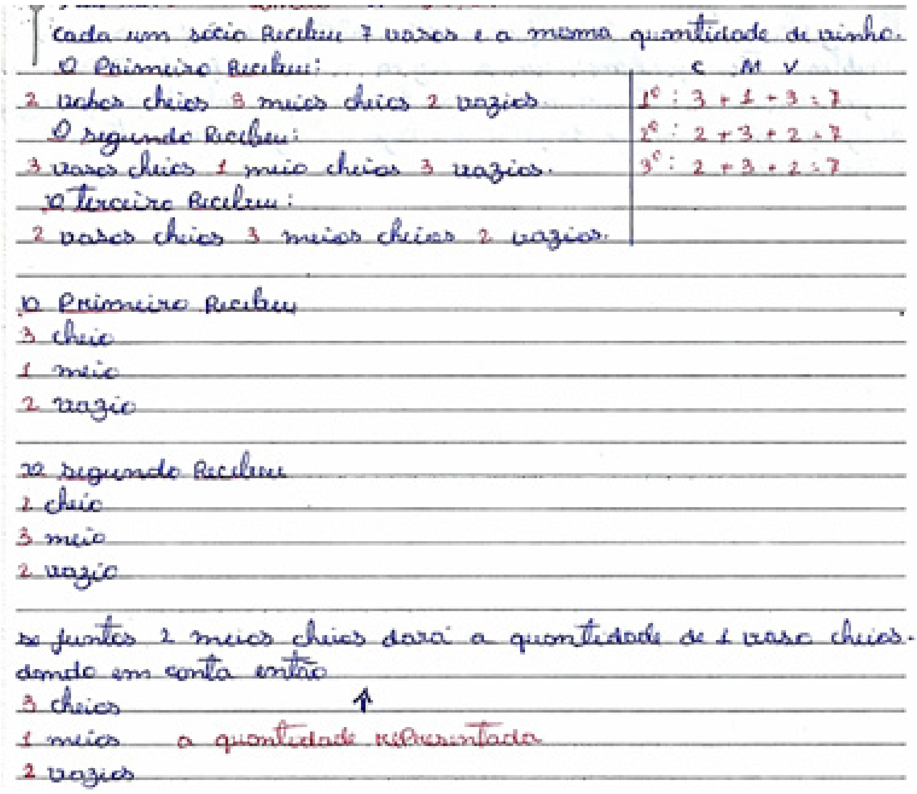
Source: Field Diary, 2023.

Figure 2 shows that the activity with Malba Tahan's book encouraged students to look for different ways of solving problems and reflect on their processes for solving the posed situation. Such processes manifest through conventional algorithms, schemes, orality, or drawings.

Another relevant aspect was the students' acceptance and analysis of the various solving strategies as valid and essential for the development of mathematical learning. This activity led students to understand that mathematical ideas can admit different forms of expression, and an expression can represent different ideas and mathematical contexts, which implies a great challenge for the mathematics teacher, "as it is an understanding that forces us to leave the comfortable position of attributing to each symbol or mathematical expression a unique meaning and, conversely, to each idea a single form of representation" (SMOLE & DINIZ, 2001, p. 123). Thus, the same mathematical model can be worked on through different semantic structures, helping the student to recognize mathematical isomorphisms through the diversity of meanings attributed to problem-solving strategies.

The emphasis on written production in mathematics classes resulted in conceptions about how students construct mathematical knowledge. Writing in mathematics classes was a mediating instrument between the subject and the object of knowledge (SMOLE & DINIZ, 2001). This is evident in Figure 3.

Figure 3: Writing Mathematical Ideas



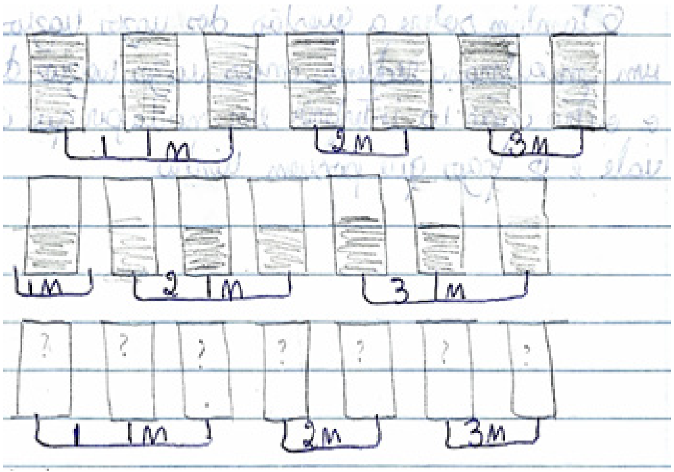
Source: Field Diary, 2023.

This register shows the importance of writing in communicating mathematical ideas. Although the student resorted to arithmetic calculations to face this situation, he emphasized the written justification of the calculations. In this sense, the activity also showed that solving a mathematical problem does not consist of simply recognizing typical situations and/or applying recurring strategies for applying algorithms, in which the only task is to develop the step-by-step procedures indicated by the teacher. However, the resolution strategies presented in this proposition were constructions that began with qualitative planning and hypothesis raising and were based on active learning (ITACARAMBI, 2010).

The students' responses showed they always look for a new way to solve the problem. Therefore, according to the students, it was not surprising to make drawings or written records to find the solution; on the contrary, this was the most recurrent strategy in the resolutions. Smole and Diniz (2001) say that in problem-solving situations, students often choose to represent their solutions based on the context or structure of the problem, which varies according to each person's confidence. One or another of their various representations came close to the operative technique, as shown in Figure 3, which does not necessarily translate into a traditional algorithm.

Reading and writing in mathematics classes provided diverse resolution (heuristic) strategies. One of the students used algebraic language to express his mathematical ideas, as shown in Figure 4.

Figure 4: Algebraic Representation for the Sharing of Casks



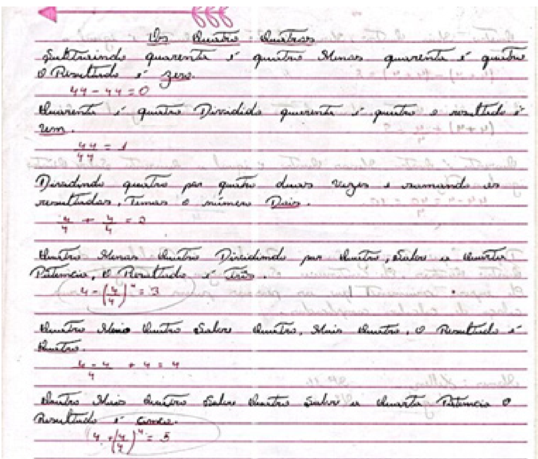
Source: Field Diary, 2023.

The structure of mathematical reasoning was as follows: 1M represented the first Muslim, 2M the second, and 3M the third Muslim, which highlighted the use of algebraic language or algebraic thinking. This strategy was used as a resource to expand understanding of the problem and as a vehicle for accessing other types of reasoning. When registering, the student externalized mathematical knowledge, revealing his interpretation of the problem and his mastery of the mathematical content of the posed activity.

Smole and Diniz (2001) believe that mathematics teachers cannot forget that one of the school's main tasks is to teach students to use reading and writing independently in all areas of the curriculum. For this reason, the research proposes that students be invited to register and communicate information and their discoveries during mathematics classes. This practice of reading, writing, and solving mathematical problems was also developed with another episode of the book *The Man Who Counted*, the problem "The Four Fours."

In this problem, "The Four Fours," students needed to form the numbers from 0 to 10 using only four fours and the fundamental operations. Therefore, they would need to resort to the properties of numerical expressions, as illustrated in Figure 5.

Figure 5: Solving the Four-Fours Problem



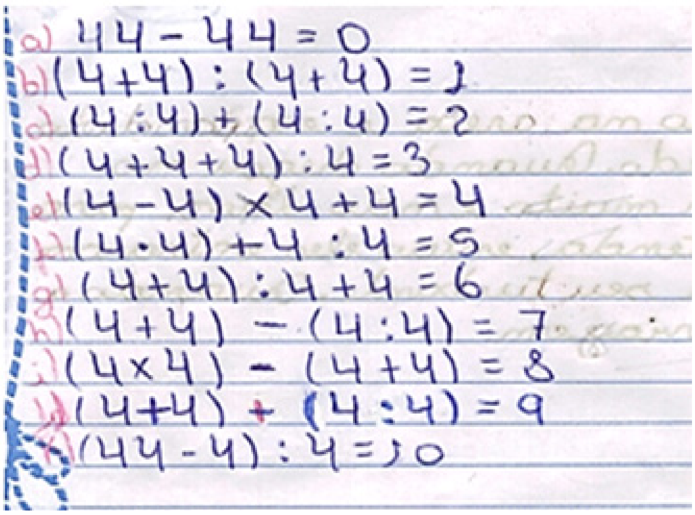
Source: Field Diary, 2023.

Figure 5 suggests, based on Onuchic (1999), that the problem-solving process enabled students to expand or deepen their mathematical skills because they used arithmetic tools that helped solve the problem and meet the conditions proposed for resolution. Furthermore, it constructed a resolution that addressed aspects related to calculation and written production that justified the development of the algorithm used. So, writing reinforced the resolution heuristic and highlighted the importance of mastering mathematical language for solving a problem.

The problem "The Four Fours" was presented in such a way that its solution was open as long as it met the given conditions, and this required students to have an autonomous and active stance in the mathematics class. This fact reinforces that "teaching based on problem solving presupposes promoting students' mastery of procedures and the use of knowledge already available" (POZO & ECHEVERRÍA, 2002, p. 30). This autonomy also helped them learn to learn, from the perspective of challenging them to find answers to the posed problems on their own.

Regarding using already available knowledge, as stated by Pozo and Echeverría (2002), the problem "The Four Fours" required students to use arithmetic tools related to numerical expressions. This encouraged the construction of mathematical knowledge related to calculating the numerical value of an algebraic expression, as this was the mathematical object that would be formalized at the end of the activities proposed to solve the problem. Therefore, the resolution process presented in Figure 6 demonstrates the students' restlessness and autonomy in seeking answers from previously acquired knowledge.

Figure 6: Numerical Expressions Created to Solve the Problem



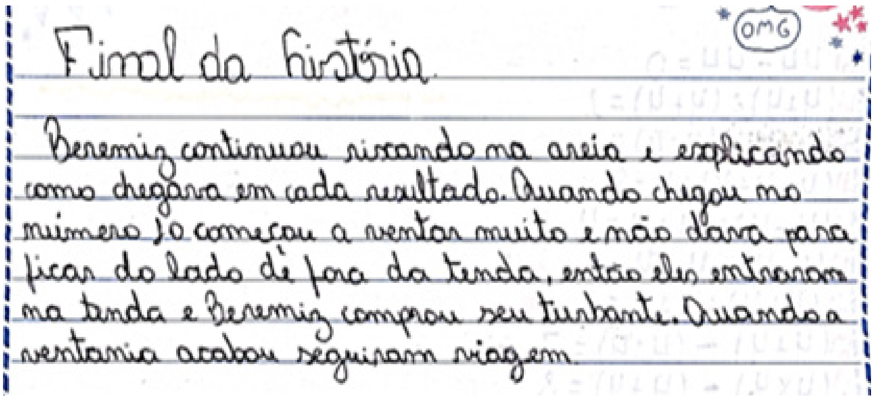
Source: Field Diary, 2023.

Figure 6 shows the result of the mathematical manipulations to find the numbers from 0 to 10. We can see that the hierarchical process in solving a numerical expression was met. The manipulation of fundamental operations (addition, subtraction, multiplication, and division) was used correctly, as were mathematical signs, such as parentheses.

Another relevant aspect of this activity is that using problem solving as a methodology for teaching mathematics awakens students to the construction of different (heuristic) strategies to solve the proposed problem. In addition, it provokes creativity and encourages students to use their knowledge already constructed and their experiences to construct new knowledge, in other words, using skills related to numerical expressions to learn how to calculate the numerical value of an algebraic expression. Therefore, from the register in Figure 6, we can infer that the problem--solving activities developed in mathematics classes focused the students' attention on the mathematical ideas expressed in the text and gave meaning to what was studied. As stated by Onuchic and Allevato (2004, p. 17), problem solving as a methodology for teaching mathematics "develops in students the ability to think mathematically" and requires them to use different strategies in different problems or the same problem.

Working with problem solving in mathematics classes, in addition to allowing the understanding of mathematical content and concepts, encouraged the practice of writing, a skill that is not restricted to the teaching of the mother tongue but which can be worked on in conjunction with the teaching of mathematical language, as exemplified in Figure 7.

Figure 7: Writing in Mathematics Classes



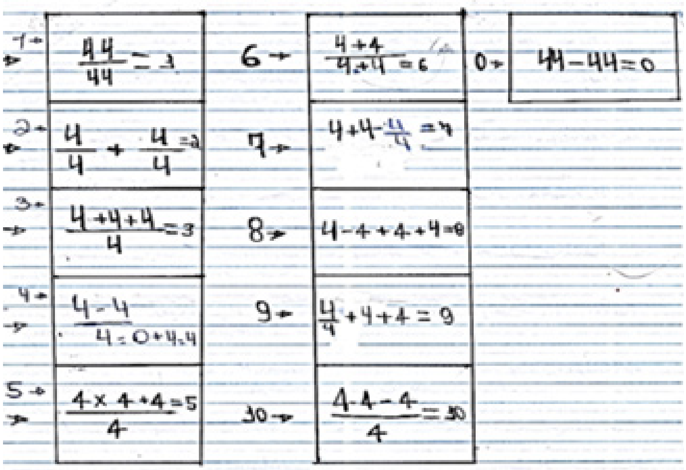
Source: Field Diary, 2023.

Figure 7 represents one of the class activities. After solving the problem, students were instructed to construct an ending for the story, as the outcome of the narrative was omitted, without prejudice to the resolution, to request, based on understanding and solving the problem, the writing of an ending to the story of Beremiz Samir (the man who counted). However, this activity did not seem very ordinary. It was strange for students to read and write in mathematics classes, as they were used to just doing calculations. However, as the classes went on, they realized that language skills (reading and writing) were critical for interpreting problems.

Onuchic and Allevato (2004, p. 20) state that "teachers who teach in this way get excited and do not want to go back to teaching in the so-called traditional way." The research results corroborate this assertion, as the teacher and students said that mathematics classes would not be the same after this experience. In her speech, the teacher felt gratified by the results presented by her students. She realized that mathematics can -and should- be worked on from a literary work and that reading and writing are not practices restricted to Portuguese language teachers but a commitment of all teachers.

Problem solving not only develops students' mathematical power, i.e., the ability to think mathematically and optimize mathematical tools, but it also increases students' understanding of mathematical content and confidence. Figure 8 shows how students could coherently use mathematical concepts and ideas to solve the problem "The Four Fours."

Figure 8: New Solutions to the Four Fours Problem



Source: Field Diary, 2023.

This activity (Fig. 8) showed, as stated by Onuchic and Allevato (2004, p. 21), that "problem solving develops the belief that students are capable of doing mathematics and that mathematics makes sense." Students showed confidence and self-esteem in solving the proposed problems, corroborating the authors above. Thus, the lesson made more sense when the teacher went to the board to formalize the concepts and mathematical ideas. The students realized the importance of highlighting the different surgical techniques and properties related to the content studied.

The classes highlighted many students' difficulty in learning mathematics, just as many teachers have in teaching. However, such difficulties can be overcome by working in a context conducive to the understanding and production of mathematical knowledge. It is enough that the class has meaning for the students.

However, we must be cautious about registering through drawings. Drawing for the sake of drawing does not constitute a form of mathematical communication, as this involves interaction with other students. Thus, for communication to happen, we had to organize activities to guarantee the mutual appreciation of students' drawings, i.e., make the drawings a vehicle for transmitting ideas. Therefore, it is important to propose situations where drawing involves discussing ideas with colleagues and exchanging ideas.

5. Final Considerations

This research, which aimed *to analyze the mathematical knowledge mobilized in the process of teaching and learning mathematics through problem solving from Malba Tahan's book The Man Who Counted*, gave rise to researchers' new perspective of the didactic-pedagogical organization of mathematics classes based on that approach as a teaching methodology.

This research mobilized teaching knowledge (pedagogical content knowledge, curriculum knowledge) related to the organization of content and the articulation of theory and practice. Therefore, teacher education courses could more solidly articulate "the knowledge produced by universities *about* teaching and the knowledge developed by teachers *in* their daily practices" (TARDIF, 2014, p. 23). If it is in these terms, the prospective teachers can have a new look at the different ways of obtaining the knowledge necessary to teach classes.

We can also infer that understanding a mathematical problem does not depend exclusively on mathematical ideas or concepts but that there is an intrinsic relationship between the mother tongue and mathematical language. Therefore, we noticed that much of what leads a student to fail when faced with a mathematical problem often depends on skills related to reading and writing. When reading a problem, students' success will depend not only on the interpretation of mathematical concepts (their logical structure) but also on their knowledge of the world, that is, their experiences. Thus, in this research, problem solving assumed the role of a contextualization instrument -as Itacarambi (2010, p. 15) says- "from the moment it proposes situations and demands a mathematical solution and leads to questioning, the research and the insertion of operations within a context."

Collective work is also worth highlighting. The collective debate on the proposed problems made research work possible. In addition to quantitative analyses, the students carried out qualitative analyses in a democratic discussion environment. The students mobilized mathematical knowledge and deepened others. They also began to encourage each other and create conjectures about the strategies used in the resolutions, but always respecting their colleagues' opinions.

Therefore, the research contributed to the understanding that difficulties in learning mathematics are often directly related to how teaching is organized and that working with problem solving can minimize these difficulties for students. Thus, the teacher can request some precautions in class, that students read the problem statements carefully, use different forms of registering solution-search procedures, understand mathematical language and verify its relationship with the mother tongue, make decisions, organize the class to collaborative work, and value and correct errors. These are some attitudes resulting from the research that mathematics teachers can adopt.

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