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Knowledge of independent events by prospective primary school teachers

Conhecimento de acontecimentos independentes por futuros professores dos primeiros anos

Conocimiento de sucesos independientes por futuros maestros de educación primaria

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Abstract

In this quantitative study, prospective primary school teachers' knowledge of independent events is investigated based on three objectives: 1) classify two given events as independent or non-independent; 2) state the definition of independent events; and 3) formulate examples of independent events. The study included 37 students attending the 2nd year of the teaching degree in basic education at a university in the north of Portugal, who solved a task about independent events. In terms of results, it should be noted that the students revealed difficulties in all objectives, more pronounced in the formulation of examples of independent events and less in the definition of independent events. Therefore, it can be inferred that many students, despite knowing the definition of independent events, could not apply this knowledge to distinguish independent events from non-independent events and formulate examples of independent events.
Keywords: Probability. Independent events. Prospective teachers. Primary school.

Resumo

Neste estudo, fundamentalmente de natureza quantitativa, investiga-se o conhecimento de futuros professores dos primeiros anos acerca de acontecimentos independentes a partir dos três objetivos seguintes: 1) classificar dois acontecimentos dados em independentes ou não independentes; 2) enunciar a definição de acontecimentos independentes; e 3) formular exemplos de acontecimentos independentes. Participaram no estudo 37 estudantes que se encontravam a frequentar o 2.º ano do curso de Licenciatura em Educação Básica numa universidade do norte de Portugal, que resolveram uma tarefa sobre acontecimentos independentes. Em termos de resultados, salienta-se que os estudantes revelaram dificuldades em todos os objetivos, mais acentuadas na formulação de exemplos de acontecimentos independentes e menos na definição de acontecimentos independentes. Depreende-se, assim, que bastantes estudantes, apesar de conhecerem a definição de acontecimentos independentes, não foram capazes de aplicar esse conhecimento para distinguirem acontecimentos independentes de não independentes e para formularem exemplos de acontecimentos independentes.
Palavras-chave: Probabilidades. Acontecimentos independentes. Futuros professores. Primeiros anos escolares.

Resumen

En este estudio, fundamentalmente de carácter cuantitativo, se investiga el conocimiento de los futuros maestros de educación primaria sobre sucesos independientes a partir de tres objetivos: 1) clasificar dos sucesos dados como independientes o no independientes; 2) establecer la definición de sucesos independientes; y 3) formular ejemplos de sucesos independientes. El estudio incluyó a 37 estudiantes que cursaban el 2 año de la Licenciatura en Educación Básica en una universidad del norte de Portugal, que resolvieron una tarea sobre sucesos independientes. En cuanto a los resultados, se destaca que los estudiantes revelaron dificultades en todos los objetivos, más pronunciadas en la formulación de ejemplos de sucesos independientes y menos en la definición de sucesos independientes. Por lo tanto, se deduce que muchos estudiantes, a pesar de conocer la definición de sucesos independientes, no fueron capaces de aplicar este conocimiento para distinguir sucesos independientes de los no independientes y formular ejemplos de sucesos independientes.
Palabras clave: Probabilidad. Sucesos independientes. Futuros maestros. Educación primaria.

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1. Introduction

From a profoundly deterministic worldview that prevailed until the 17th century, we have since seen an increasing influence of uncertainty in our lives. Currently, many of the decisions we face daily involve uncertainty, whether in the context of politics, finance, or health.

From the mid-20th century onwards, uncertainty began to be recognized as necessary, which led to its introduction in secondary schools (from 10th to 12th grade, 16-18 years old) and, in the early 1990s, in basic education (from 1st to 9th year, 6-15 years old). Recently, several researchers have been advocating the teaching of probabilities and statistics in the first grades of school (Batanero, 2013; Borovcnik; Peard, 1996) and even in early childhood education (Alsina, 2021; Batanero et al., 2021; Nikiforidou & Pange, 2010).

Probability and statistics are currently part of mathematics programs in basic education (Ministério da Educação, 2021) and secondary education (Ministério da Educação, 2023) in Portugal. In Brazil, after the release of the National Common Curriculum Base (Brasil, 2018), we are also witnessing the deepening of the study of probabilities and statistics in primary and secondary education, which highlights the exploration of statistical investigations and the encouragement of the use of digital technologies, such as spreadsheets (Fernandes & Diniz, 2022).

Now, deepening the teaching of probabilities and statistics in schools requires that pre-service and in-service teachers acquire knowledge that allows them to implement adequate teaching, as recent studies show that prospective Portuguese teachers in the first years of school experience difficulties in probabilities (Fernandes, 2022; Fernandes & Barros, 2021; Fernandes & Oliveira Júnior, 2023;) and in Statistics (Fernandes, 2021, Fernandes, & Freitas, 2019).

Given the above, this study focuses on the theme of probabilities and the performance of prospective first-year teachers in independent events, based on the following objectives: 1) classify two given events as independent or non-independent; 2) state the definition of independent events; and 3) formulate examples of independent events.

After alluding to the importance and purposes of the study –aspects previously addressed–, we discuss the theoretical framework focused on independent events. We then describe the methodology used and present the results. Finally, we summarize the main conclusions and draw some implications for probability teaching.

2. Theoretical framework

Carrying out a random experiment produces any possible results, which define the so-called space or universe of results. Therefore, we can define the set, which is the space of results and any of its subsets. Any of these subsets defines an event associated with the random experience.

Different types of events can be defined based on different attributes: certain, possible (but not certain), impossible, incompatible (or disjoint or mutually exclusive), complementary, and independent events. Certain, possible (but not certain), and impossible events are defined through subsets of the outcome space, i.e., the outcome space itself (certain), a different and non-empty subset of the outcome space (possible but not certain), and the empty subset (impossible). Therefore, these are simple events that students in Portugal begin to study in their third grade of schooling (Ministério da Educação, 2021).

The definition of incompatible, complementary, and independent events involves two events that fulfill specific attributes: their intersection is the impossible event (incompatible), their intersection is the impossible event, and its union is the certain event (complementary) and the probability of the occurrence of one of the events does not depend on the occurrence of the other (independent).

Thus, these events are more complex and are studied in more advanced school grades: incompatible and complementary events are introduced in the 9th grade (Ministério da Educação, 2021), and independent events are introduced in the 12th grade (Ministério da Educação, 2023).

Among the most complex events, Martins (2017) draws attention to the frequent confusion between incompatible and independent events, clarifying that two events cannot be incompatible and independent simultaneously unless one of them is the impossible event. To overcome this confusion, the author suggests considering that these concepts assume different relationships: the incompatibility of events is a property of events; it is not necessary to define any probability, and the independence of events is dependent on the probability model established in the outcome space where events are defined.

In the study of the independence of events, which is the focus of the present study, the concept of conditional probability plays an important role and can be used to verify whether or not two events are independent. In this case, we say that two events A and B are independent if the probability of the occurrence of any of them is not affected by the occurrence of the other, i.e., when one of the two relationships is verified: $P(A \mid B)=P(A)$, with $P(B)\neq 0$, or $P(B \mid A)=P(B)$, with $P(A)\neq 0$, thus constituting an alternative to using the relationship $P(A\cap B)=P(A)\times P(B)$ to verify the independence of events. Huff (1971, cited by Hawkins et al., 1992) proposed the definition of independent events based only on conditioned probabilities, stating that two events A and B are independent if $P(A\mid B) = P(A \mid \bar{B})$, where $\bar{B}$ is the complementary event of B.

Fernandes et al. (2014) questioned prospective teachers of the early years about probability conditioned on four items involving replacement and non-replacement in the extraction of two balls from a bag and the choice at random of two people from a group of men and women. Overall, on average, 56% of correct answers were obtained for each item, which is considered a reasonable performance. In general, students performed better on drawing balls out of a bag than on choosing people from the group, with P(1st person be a man|2nd is a woman) proving to be very difficult, which is explained by the time-axis reversal, as we wrongly believe that an event carried out later cannot affect one carried out before (Contreras et al., 2013).

In the case of independence, Fischbein et al. (1991) asked students from the 4th to the 8th grade of schooling without probability instruction how likely it was that they would get three heads in three consecutive tosses of a coin or the simultaneous toss of three coins. A third of the students said that the probability was not the same, which resulted from the belief that they were more likely to get three sides heads in three consecutive tosses of a coin than in the simultaneous toss of three coins. From interviews, the authors concluded that students believed the individual could control the results of tossing a coin. This belief is incompatible with the independence of events since the probability of obtaining heads in each toss is constant and equal to 1/2.

Correia and Fernandes (2014), after obtaining the sequence CCCCC in five consecutive coin tosses, asked 9th-grade students whether they were more likely to obtain face C (heads) or side E (shield) on the sixth toss of the coin or whether they would be equally likely to get either side of the coin. In the task, 89.7% of students stated that it is equally likely to obtain any side of the coin, highlighting, in terms of justification, the equal probability of obtaining each side of the coin, the possibility of obtaining any side of the coin, the coin being balanced, and the trials are independent. In the wrong answers, the answer that was more likely to get the head was highlighted, with most of these students saying that it would be more likely because the result was always heads. Thus, these students joined the so-called positive recent effect (Fischbein, 1975), which states that after obtaining a result several times in a row, they would be more likely to obtain the same result on the next flipping. The results led authors to think that students have an intuitive substrate in line with the possibility of developing a more formal approach at school.

In a more recent study, Fernandes and Barros (2021) asked prospective early years teachers to define pairs of examples of disjoint, complementary, and independent events in the random experiment of rolling a die twice in a row. In the case of independent events, the focus of our study, they found that students had great difficulty establishing the pair of independent examples, with only 23% answering correctly.

3. Methodology

The purpose of the study is to investigate the knowledge of prospective early years teachers about independent events based on three objectives: 1) classify two given events as independent or non-independent; 2) state the definition of independent events; and 3) formulate examples of independent events. Thus, the study of the concept of independent events, based on the definition of the concept, the classification of given examples, and the formulation of examples, allow students to investigate the understanding of the concept in greater scope (Skemp, 1993).

To comply with the objectives of the study, we conducted a quantitative and descriptive investigation. In this type of investigation, a pre-existing reality is analyzed, such as knowledge about various aspects of independent events, without exercising any control and using rigorous methods (McMillan & Schumacher, 2014).

Thirty-seven students attending the 2nd year of the teaching degree in basic education at a university in the north of Portugal participated in the study. After completing their degree, these students can attend and complete a master's degree in teaching, which provides them with professional qualifications to teach at the early childhood level or from the 1st to the 6th grades. Upon entering university, students brought mathematical knowledge they had acquired during their secondary school in professional, humanistic, or scientific-technological courses.

The data used in the study are the students' resolutions when answering a questionnaire with several tasks about different types of events. Here, we deal with the one that concerns independent events, presented in Figure 1. We applied the questionnaire after the students had attended the unit Probabilities and Statistics and ensured students' anonymity and confidentiality in any publication involving these data.

Figure 1: Task proposed to students

1. In each following item, check whether events A and B are independent, being:
a. A: "get head face on the 1st flipping" and **B**: "get the shield face on the 2nd flipping", in the random experiment of tossing a coin twice.
b. A: "get two even faces" and **B**: "get two faces greater than 5", in the random experience of rolling twice a die numbered from 1 to 6.
2. Regarding a random experiment,
a. Define when two events **A** and **B** are independent.
b. Define two **independent** events **A** and **B**, different from those defined in Question 1.

Fonte: Elaboração do autor (2022)

Figure 1 shows that the task consists of two questions, Question 1 and Question 2, each with two items. In the two items of Question 1, two events are given, and students are asked whether or not they are independent. The events in 1a relate to the experience of flipping a coin in the air twice, and in 1b, the experience of rolling a die twice. In Question 2, in 2a, students must state the definition of independent events, and in 2b, they must define two independent events distinct from those given in Question 1.

Finally, in data treatment and analysis, we studied students' answers about classifying events as independent and non-independent, the explanations for this classification, and the definition and exemplification of independent events. In all cases, frequencies of types of events (independent and non-independent) and explanations of answers (correct, partially correct, and incorrect) were determined using tables to summarize this information. Additionally, to deepen the understanding of the analysis process, examples of student responses are presented, identified by the letter S

4. Presentation of results

This section presents the results of the study, organized according to each of the settled objectives: classifying two given events as independent or non-independent, stating the definition of independent events, and formulating examples of independent events.

4.1. Classifying events as independent or non-independent

This objective includes the two items, 1a and 1b, from Question 1. Next, students' resolutions on these two items are analyzed.

Item 1a. In this item, students should state that the events A and B are independent using the definition of independent events. Table 1 presents students' different types of answers when answering Item 1a.

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Tabela 1: Frequência (em %) de estudantes segundo o tipo de resposta no item 1a

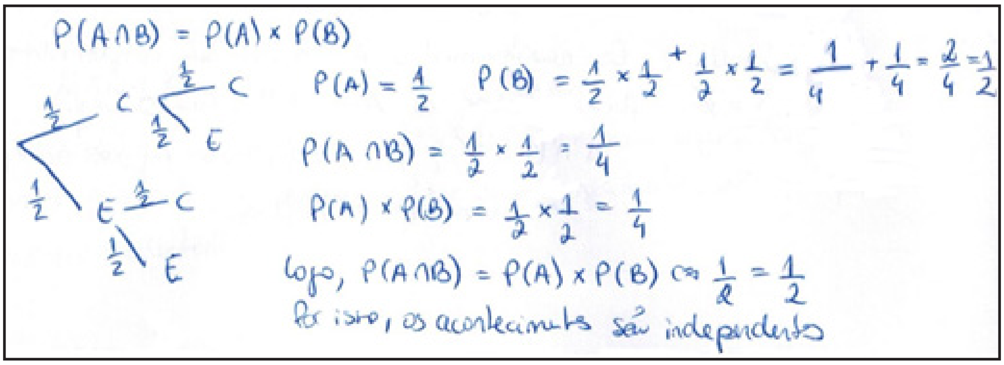
Answer type	Attendance (in %)
Correct	14 (38)
Partially correct	14 (38)
Incorrect	7 (19)
No answer	2 (5)

Source: Developed by the author (2023)

Table 1 reveals that less than half of the students answered correctly, and many others presented partially correct answers. Around one in five students gave wrong answers, and only two did not answer. Next, the ideas on which the students based their answers are presented.

Regarding correct answers, three students showed that the relationship P(A∩B)=P(A)×P(B) occurs, and the remaining 11 stated that no event A or B depends on, conditions, or affects the probability of the other. Figure 2 presents an example of the first type of resolution.

Figure 2: S9's resolution of Item 1a

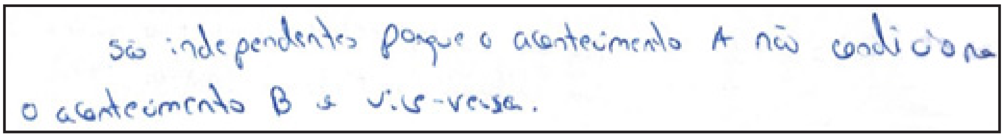


Source: Developed by the student (2022)

In his resolution, S9 determined the respective probabilities. He found that the relationship P(A∩B)=P(A)×P(B) is verified, although he failed to write 1/2=1/2 instead of 1/4=1/4, and concluded that the events are independent.

Concerning independence established by the definition that no event depends on, conditions, or affects the probability of the other, Figure 3 presents an example of this type of resolution.

Figure 3: S20's resolution of Item 1a



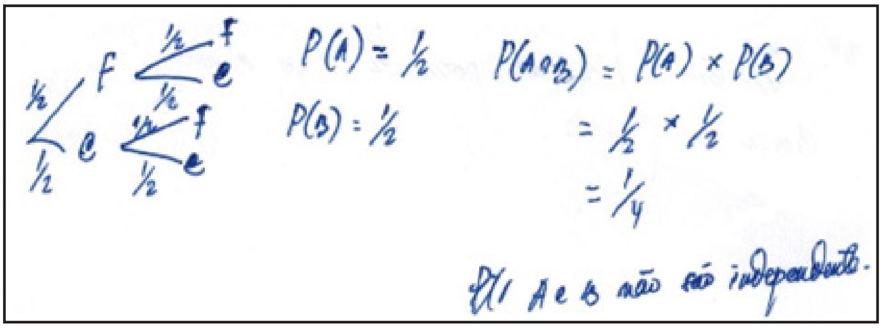
Source: Developed by the student (2022)

S20 states that the events do not affect each other, probably using the knowledge and experience developed in the Probability classes. In this regard, it is common in classes on probabilistic independence to point out that in two tosses of a coin or dice and the extraction with replacement of two objects from a bag the probability of obtaining any result is independent of the result obtained in the other toss or extraction.

In the partially correct answers, eight students did not provide any explanation for their answer, and the remaining six referred to the space of results, the replacement of the coin, the equality of probabilities (P(A)=P(B)), or tossing a coin.

Finally, in the incorrect answers, one student did not justify the answer, and the remaining six stated the dependence of the events, determined incorrect probabilities, and incorrectly applied the formula P(A∪B)=P(A)+P(B) or the formula P(A∩B)=P(A)×P(B). Figure 4 shows an example of this last type of resolution.

Figure 4: S3's resolution of Item 1a



Source: Developed by the student (2022)

In this resolution, S3 determines the probabilities P(A) and P(B) using a tree diagram, assumes the independence of events A and B (which he was asked to show), calculates P(A∩B), and incorrectly concludes that the events are not independent.

Item 1b. In this item, students should show that events are not independent. To do so, and using the definition of independent events, they should conclude that this definition does not apply to the given events. Table 2 shows students' different types of answers when responding to Item 1b.

Table 2: Frequency (in %) of students according to the type of response in Item 1b

Answer type	Attendance (in %)
Correct	2 (5)
Partially correct	18 (49)
Incorrect	12 (32)
No answer	5 (14)

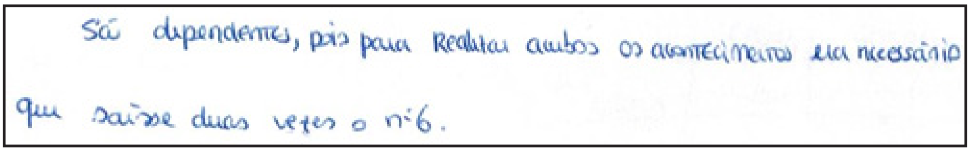
Fonte: Elaboração do autor (2023)

Item 1b proved to be more difficult than Item 1a, with fewer students now providing correct answers and more students providing incorrect or no answers. About half of the students gave partially correct answers, followed by those who gave incorrect answers, and finally, only two gave the correct answer. We analyzed students' ideas underlying their resolutions as follows.

In the case of hits, both students resorted to the relationship P(A∩B)=P(A)×P(B) to conclude that the events are not independent. Compared to Item 1a, fewer students hit this one. In this item, it was not appropriate to state that the probability of any of the events is not dependent, conditioned, or affected by the occurrence of the other, as the aim was to explain the dependence (not the independence) of the events. Therefore, unlike independent events, students' lack of knowledge and experience about dependent events may explain their poor performance in this item.

In the partially correct answers, as in Item 1a, eight students did not present any explanation for their answer. Four focused on the face higher than five, that is, on face six, as exemplified by the resolution in Figure 5. Two considered just the roll of one die, and the remaining four presented different explanations.

Figure 5: S36's resolution of Item 1b

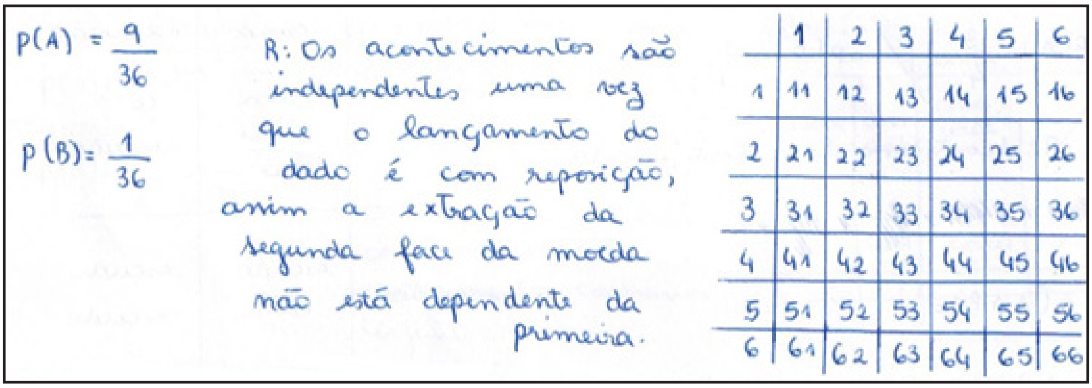


Source: Developed by the student (2022)

Student S36's resolution revealed that the dependence of the events results from obtaining two sides of six in both throws of the die, which is exactly event A∩B. Therefore, the student did not fully explain his answer.

In the incorrect answers, five students did not offer any explanation about their answers. In the remaining seven responses, students considered that one event does not make the other impossible, incorrect probabilities were determined, or neither event is conditioned by the other. Figure 6 shows an example of this last explanation.

Figure 6: S28's resolution of Item 1b



Source: Developed by the student (2022)

In his solution, S28 correctly determines the probabilities P(A) and P(B) using a double-entry table but does not calculate P(A∩B). He then considers that the dice roll is done with replacement, which does not apply, and wrongly concludes that the events are independent.

4.2. State the definition of independent events

This objective only includes Item 2a of Question 2, of which the students' resolutions are then analyzed. It asks students to define when two events are independent. Therefore, given events A and B, students were expected to state that relationship $P(A \cap B)=P(A)\times P(B)$ should be verified or one of the relationships $P(A|B)=P(A)$, with $P(B)\neq 0$, or $P(B|A)=P(B)$, with $P(A)\neq 0$. Alternatively, students can present a verbal definition, stating that the occurrence of either event does not affect the probability of the other occurring. Table 3 shows the different types of answers students gave in Item 2a.

Table 3: Frequency (in %) of students according to the type of response in Item 2a

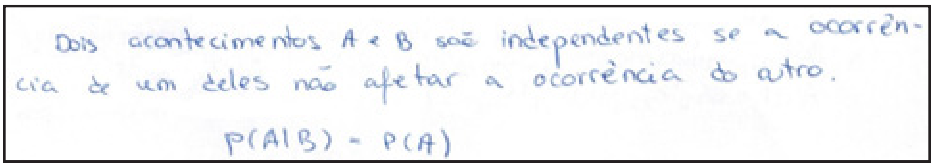
Tipo de resposta	Frequência (em %)
Correta	32 (86)
Parcialmente correta	—
Incorreta	4 (11)
Não resposta	1 (3)

Source: Developed by the author (2023)

Table 3 indicates that students were more successful in defining independent events (Item 2a) than classifying events as independent or non-independent (Items 1a and 1b). Specifically, almost all students gave correct answers, there were no partially correct answers, and few gave incorrect answers or did not answer at all. Next, we present the analysis of the definitions mentioned by the students in solving Item 2a.

In the hits by almost all students, five defined independent events based on relationship $P(A \cap B)=P(A)\times P(B)$, one resorted to relationship $P(A|B)=P(A)$ and two to the two previous relationships. Finally, most students, 24 in total, reported that the occurrence of one of the events does not affect the probability of the other. Figure 7 exemplifies the explanation based on the second relationship.

Figure 7: S25's resolution of Item 2a

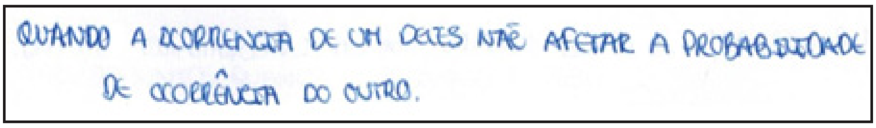


Source: Developed by the student (2022)

Student S25 defines independent events in verbal and symbolic language in his resolution. Thus, it begins by mentioning that two events A and B are independent if the occurrence of one of them does not affect the occurrence of the other and then establishes the symbolic relationshipP(A|B)=P(A).

Figure 8 provides an example of a verbal-only definition of independent events.

Figure 8: S17's resolution of Item 2a



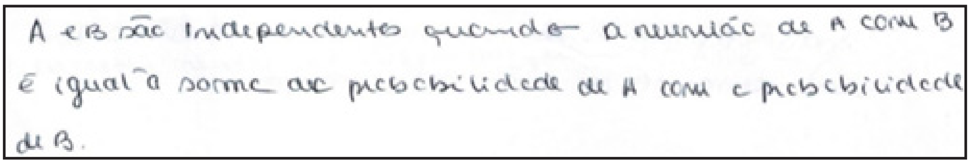
Source: Developed by the student (2022)

In his definition, S17 says that events are independent when the occurrence of one does not affect the probability of the other. This constitutes a precise definition, as what is implicit is the verification of one of the relationships P(A|B)=P(A) or P(B|A)=P(B).

However, verbal definitions like the one presented by student S17 are limited in their operationalization. In situations involving throwing coins and dice or extracting objects with replacement, knowledge and experience allow us to explain the independence of events. However, it may be necessary to resort to one of the relationships between probabilities in less usual situations to verify whether the events are independent.

Finally, in incorrect answers, students presented different ideas, such as *"For it to happen, it is not necessary for the other to happen," "Depend on themselves,"* or refer, explicitly or implicitly, to the idea of disjoint events. Figure 9 shows an example of disjoint events.

Figure 9: S29's resolution of Item 2a



Source: Developed by the student (2022)

Student E29 seems to focus on the relationship P(A∪B)=P(A)+P(B), which requires that events A and B are disjoint. However, the statement is not entirely clear when it refers to the event "union of A with B" and not, as it should be, "the probability of the union of A with B ."

4.3. Formulate examples of independent events

Only Item 2b of Question 2 is part of this objective, of which we analyze the students' resolutions. Students are asked to define two independent events different from those given in Question 1. Therefore, as the item is open, students must present different solutions.

Table 4 shows the different types of answers given by students when answering Item 2b

Table 4: Frequency (in %) of students according to the type of response in Item 2b

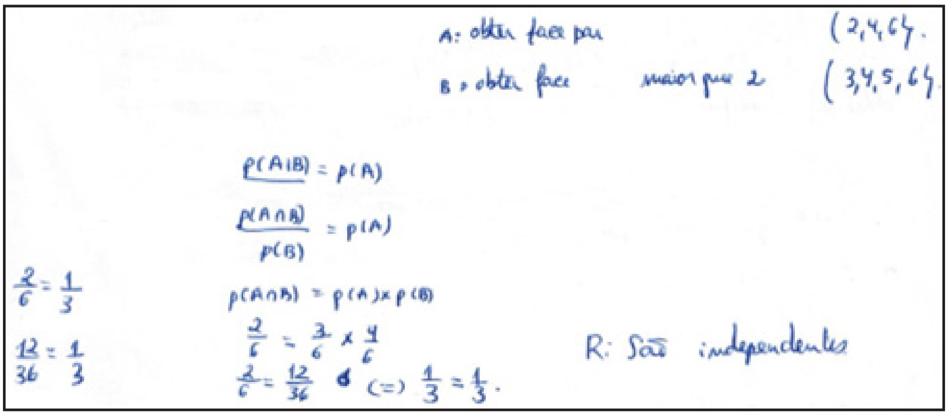
Answer type	Attendance (in %)
Correct	4 (11)
Partially correct	10 (27)
Incorrect	21 (57)
No answer	2 (5)

Source: Developed by the author (2023)

Table 4 shows that more than half of the students gave incorrect answers, followed by partially correct answers. Only four students gave correct answers, and two did not answer. We concluded, therefore, that this item was the one that caused the students the most difficulties. Next, we analyze the students' ideas on which they based their resolutions.

In the correct answers, which are only four, students considered the experience of throwing a die or drawing balls from a bag. Figure 10 shows the experience of throwing a dice.

Figure 10: S24's resolution of Item 2b

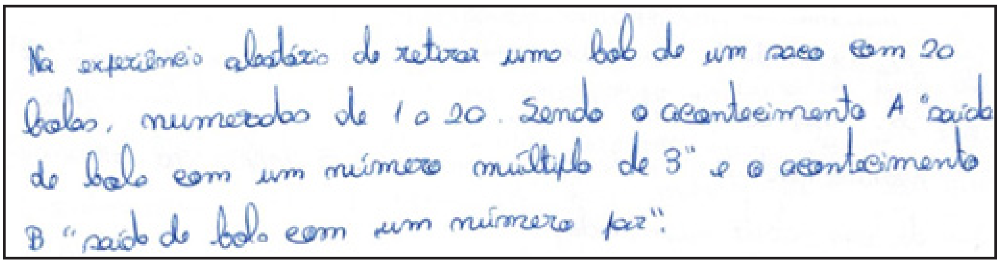


Source: Developed by the student (2022)

In the dice-throwing experience, S24 considered two events A and B, then determined probabilities P(A), P(B), and P(A∩B) and, finally, showed that relationship P(A∩B)=P(A)×P(B) verifies without, however, defining event A∩B. In the remaining three resolutions, students considered experiences of drawing balls/cards with replacement, which was seen as sufficient, even without verifying the independence relationship. Analogously to the case of throwing coins and dice, r*eplacement* was considered sufficient knowledge and experience to justify independence.

In the partially correct answers, no student verified the independence condition nor mentioned replacement in the experience of drawing balls from a bag. Figure 11 shows an example of this type of resolution.

Figure 11: S11's resolution of Item 2b

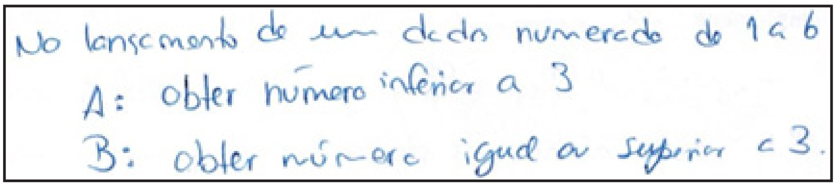


Source: Developed by the student (2022)

S11 establishes two events of the random experiment of drawing a ball from a bag with 20 balls numbered from 1 to 20. Although the events are independent, the student does not prove they are truly independent.

Finally, in the incorrect answers, nine students did not provide any explanation for their answers, and five mentioned the experience of throwing a die. The resolution of this item seems to have been based on the definition of incompatible or complementary events, as exemplified in Figure 12.

Figure 12: S6's resolution of Item 2b



Source: Developed by the student (2022)

In his resolution, S6 defined two events A and B, complementary in the experience of throwing a dice, which are not independent because A∩B is the impossible event, and both A and B are not impossible events.

Of the remaining students, four present unintelligible resolutions, and three do not specify concrete events or establish coinciding events. Note that in the case of coinciding events, we concluded that P(A|A)=1 for any value of P(A)≠0, where A is independent of A only when A is the certain event.

5. Discussion and conclusion

When classifying events as independent and non-independent, including Items 1a and 1b, most students gave correct or partially correct answers, with very few (only two) giving correct answers to Item 1b. In hits, students explained their answers based on one of the relationships that define independent events or that no event depends on, conditions, or affects the other. This last explanation is possibly due to the students' better performance in Item 1a, as in Item 1b, it does not apply because the events are dependent. In the partially correct answers, many students did not explain their answers.

We found that almost all students gave correct answers to the definition of independent events evaluated in Item 2a. In this item, the hits were explained through relationships that define independent events, or verbally, alluding to the fact that the occurrence of one event does not depend on or affect the probability of the other.

Finally, when formulating examples of independent events evaluated in item 2b, most students gave incorrect answers, thus constituting the least achieved objective of the study. Among the correct and partially correct answers, the correct answers were much fewer, and in total, these two types of answers amounted to little more than 1/3 of all answers. For the hits, students considered experiences of throwing a die or drawing balls from a bag, and in partially correct answers, no student verified the independence of the events nor mentioned replacement in the experience of drawing balls from a bag. Thus, the difficulties students experience in exemplifying independent events confirm those observed in a previous study by Fernandes and Barros (2021) with prospective teachers of the early years, too.

Studies on other types of events in which the same prospective teachers of the early years participated also observed students' better performance in defining independent events, namely in defining disjoint events (Fernandes, 2022) and complementary events (Fernandes, 2024). Now, students' more significant success in defining different types of events in this study and the two others mentioned means that this is a relatively robust result, which in turn challenges the teaching and learning of such concepts.

More specifically, students' ability to define disjoint, complementary, or independent events does not imply that they can classify given events and give examples of events of these different types, as demonstrated in this study and the others mentioned before. Students' weaknesses most likely originate from routine learning, which is based only on memory and is not significant (Ausubel et al., 1980).

To overcome the students' limitations observed in the study, it seems to us that the exploration of tasks simultaneously addressing the classification, definition, and exemplification of independent events, as happened in the present study, could contribute to more significant learning of the students, prospective teachers of the early years, in which the application of the definition of events that are independent on the classification of given events and the formulation of examples is explicitly highlighted.

On the other hand, regarding students' difficulties in distinguishing incompatible, complementary, and independent events, we suggest that common attributes and those that distinguish them be emphasized in teaching these concepts. Thus, given events A and B, attribute $A \cap B = \emptyset$ is common to incompatible and complementary events, and complementary events are distinguished by also requiring attribute $A \cap B = U$, in which U is the results space. If one of the probabilities P(A) or P(B) is null, the events are independent (Martins, 2017), while if $A \cap B = \emptyset$ and P(A) and P(B) are non-null, the events are not independent, i.e., incompatible and complementary events are not independent as long as $P(A) \neq 0$ and $P(B) \neq 0$.

Given the difficulties prospective teachers experienced, deepening their education on different types of events, particularly independent events, is essential. Such in-depth study is essential for students who did not attend scientific-technological courses in secondary education, as the mathematical content in these courses is very superficial. Therefore, it is plausible that students' mathematical education, acquired throughout secondary education, interferes with the difficulties students experienced when dealing with independent events. That is, students with a more robust or fragile previous mathematics education present less or more difficulties in independent events. Therefore, carrying out a future study that breaks down students' difficulties according to the mathematical education received in the courses they took in secondary education will allow us to delve deeper into the origin of their difficulties.

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