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Visual geometric thinking — an investigation in a short course/workshop

Pensamento visual geométrico — uma investigação em um minicurso/oficina

Pensamiento geométrico visual: una investigación en un curso/taller corto

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Abstract
The research, of a qualitative nature, sought to answer the following question: how can participants in a didactic workshop develop geometric visual thinking based on activities with folds on an A4 sheet involving fundamental elements of euclidean geometry? To this end, the general objective was to analyze the geometric visual geometric thinking of participants in a didactic workshop when carrying out activities with folds on a sheet of paper and identifying geometric elements contained in these folds. The results showed a limitation in the visualization of straight lines, half-lines, straight segments, reflections, polygons and polygonal regions, which characterizes a lack of developing visual geometric thinking in the teacher of training of mathematics teachers.
Keywords: Geometric folds. Geometric entities. Visualization.

Resumo
A pesquisa, de cunho qualitativo, buscou responder ao seguinte questionamento: como participantes de uma oficina didática podem desenvolver pensamento visual geométrico a partir de atividades com dobras em uma folha A4 envolvendo elementos fundamentais da geometria euclidiana plana? Para isso, delineou-se o objetivo geral: analisar o pensamento visual geométrico de participantes de uma oficina didática ao realizarem atividades com dobras em uma folha de papel e identificar elementos geométricos constantes dessas dobraduras. Os resultados mostraram que há uma limitação na visualização de retas, semirretas, segmentos de retas, reflexões, polígonos e regiões poligonais, o que caracteriza a falta de um desenvolvimento do pensamento visual geométrico na formação de professores que ensinam matemática.
Palavras-chave: Dobras geométricas. Entes geométricos. Visualização.

Resumen
Esta investigación cualitativa buscó responder a la siguiente pregunta: ¿cómo pueden los participantes de un taller didáctico desarrollar el pensamiento visual geométrico a partir de actividades con pliegues en una hoja A4 involucrando elementos fundamentales de la geometría plana euclidiana? Para ello se trazó como objetivo general analizar el pensamiento visual geométrico de los participantes de un taller didáctico al realizar actividades con pliegues en una hoja de papel e identificar los elementos geométricos contenidos en dichos pliegues. Los resultados mostraron que existe una limitación en la visualización de líneas rectas, semi-rectas l, segmentos rectas, reflexiones, polígonos y regiones poligonales, etc. lo que caracteriza una falta de desarrollo del pensamiento visual geométrico en la formación de profesores que enseñan matemáticas.
Palabras clave: Pliegues geométricos. Entidades geométricas. Visualización.

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1. Introduction

Euclid's geometry privileged definitions, propositions, and theorems; however, some 19th--century mathematicians introduced innovations, such as the two pioneering Non-Euclidean, so to speak, the spherical and the hyperbolic geometries. They did not stray too far from previous aspects, although with new approaches, such as exploring the spherical geometry on the Earth or a styrofoam ball and the hyperbolic geometry on a pseudosphere, that is, a surface obtained by rotating the tractrix around its axis. These creations may have motivated other geometries, such as the taxicab and the fractal, which can be visually perceived around human beings.

Currently, visualization skills are a topic that has guided our research concerns because they can favor geometry teaching and learning at different educational levels. Such ability is not necessarily innate and can be developed from the beginning of schooling, as can be seen in Piaget and Inhelder's (1993, p. 22-21) statement:

> The first two stages of the development of space are characterized by the lack of coordination of the various sensory spaces with each other. In particular, due to the lack of coordination between vision and prehension, the visual and tactile-kinesthetic spaces are not yet linked into a single whole, etc.

Currently, researchers have focused on the development of mathematical visual thinking and, particularly for our research, visual geometric thinking as well, considering teachers' and students' difficulties in the area, as, often, "teaching through theorems, definitions, and application of formulas" persists.

The above justifies this research, which analyzes how a group of teachers and prospective teachers participating in a workshop at a regional event in southern Brazil, proposed by the author of the present article, carry out an exploratory teaching activity through folds in a simple sheet of paper and visualize basic elements of plane euclidean geometry such as lines, half-lines, line segments, parallelism, perpendicularity, reflecting on them until they obtain a rectangle (or square) and respective regions limited by such elements.

The following research question derives from those concerns: How can participants in a didactic workshop develop visual geometric thinking through tasks involving folds on an A4 sheet involving fundamental elements of plane euclidean geometry?

2. Theoretical framework

This item presents some theoretical foundations for visual thinking in mathematics, focusing on geometry.

At first, we searched in the CAPES Catalog of Theses and Dissertations in 2023, available at https://catalogodeteses.capes.gov.br/catalogo-teses. We used the descriptor "visual thinking" to find works containing this term. The search yielded 37 works, 17 master's dissertations, and 14 doctoral theses.

After that, the period outlined was from 2020 to 2023, which resulted in 13 works, seven master's dissertations, and six doctoral theses. Of these, only two had the descriptor "visual thinking" in the title, while the others had it only in the abstract.

The following restriction was the knowledge area "sciences and mathematics teaching," with only three works remaining: one master's dissertation and two doctoral theses. This search anchored the idea of the scarcity of Brazilian works on the theme; nevertheless, being a promising area of research, it has led an international group from Latin America to organize an event called *Visual Thinking in Mathematics*, which is holding its 4th edition this year.

Thus, visual thinking in mathematics education can be acknowledged as an enhancer for teaching and learning, as it allows us to understand non-simple topics, i.e., more complex ones, as Roam (2012, p. 260) indicates. He says: "The purpose of the drawings is not to eliminate words, but rather to replace as many of them as possible so that we only use those truly important [...]. This way, we have more time to talk about their meaning." In this sense, the research presented here sought to elucidate how thoughts about geometric elements obtained concretely by folding paper in synchrony can be translated into words (concepts) to arrive at such elements without using drawn figures.

In Da Mata's research (2020, p. 6), respondents used visual thinking, reporting that "besides being a great resource for observing students' development, it also allows them to develop autonomy." According to the author's conclusion, it is essential to observe the difficulties presented by the participants in establishing relationships between the objects marked on the sheet and concepts and relationships corresponding to these representations in the present research. Therefore, one of the characteristics of visual thinking is that it is a technique for constructing visual representations for mathematics (geometric) education, whether obtained through sight or mind.

Regarding visualization, Leivas(2009, 111) defines it as: "[...] a process of forming mental images, to construct and communicate a specific mathematical concept, to assist in solving analytical or geometric problems." Therefore, thought is a complex combination of words, images, scenarios, colors, and even sounds or music, according to Creswell (2013, cited by Anwar & D JuandI, 2020), and is also a solution to problems by using ideas and other elements, agreeing with what was said by Gilmer (1970), cited by the same author, so that it is a form of mental activity or information processing, which indicates that it is in tune with AAA's (2009) concept.

Authors such as Anwar and D Juandi (2020, p. 5) define levels of geometric thinking in geometry (Chart 1).

Chart 1: Characteristics of visual thinking level in geometry

Visual thinking level	Characteristics
Non-visual	- students do not involve visual thinking in solving geometric problems; - students tend to solve problems using symbolic representations; - students are still in the invalid category in solving geometry problems;
Local visual	- students already involve visual thinking; - students have not been able to distinguish the relationship between different images in geometry; - Students use symbolic representation correctly; - students can solve geometric problems.
Overall visual	- students already involve visual thinking; - Students can distinguish relationships between images, begin to recognize the characteristics they observe, and mention the regularities found in the images they make or observe; - The students correctly used symbolic representation; - Students solve geometry problems correctly.

Source: Adapted from Anwar and D JuandI (2020, p. 5)

Understanding the importance of these levels is central, as it makes it possible to begin developing visual geometric thinking from the initial years of schooling. In this direction, Santiago and Ciríaco (2023) bring a literature review at the National Meeting on Mathematics Education (Encontro Nacional de Educação Matemática - ENEM) focusing on geometry treatment based on topological notions, which should be the outset for the development of visual geometric thinking, as children usually concretely explore their environment, which can start the visual process that leads to mental constructs of geometric entities. Piaget and Inhelder (1993) reinforce this topological aspect.

Santiago and Ciríaco (2023) indicate that "[...] spatial questions are brought up by children, intuitively, when they arrive at the early childhood education environment, and this should be the onset of the exploration of geometric thinking" (p. 6). In this regard, Leivas et al. (2023) characterizes visual geometric thinking based on the articulation between imagination, intuition, visualization, and creativity and indicates several possibilities of actions that can lead to such development, including actions to be developed from basic to higher education.

Machado et al. (2022) deploy relevant aspects of visual and verbal skills from the perspective of Van Hiele's theory, which advocates levels of development of geometric thinking. Chart 2 illustrates what follows.

Chart 2: Geometry skills.

Skills	Identification	Analysis	Order	Deduction	Rigor
Visual	Identifies different figures in a drawing. Identifies tagged information in a figure.	Perceives the properties of a figure as part of a larger figure.	Identifies interrelationships of different types of figures. Identifies common properties of different types of figures.	Identifies information about a figure to deduce other information.	Identifies unjustified assumptions made through the use of figures. Conceives related figures in various deductive systems.
Verbal	Associates the correct name with a given figure. Interprets sentences that describe figures.	Accurately describes various properties of a figure.	Defines words precisely and concisely. Formulates sentences showing interrelationships between figures.	Understands the difference between definition, postulates, and theorems. Identifies what is given and what is asked to find or do.	Formulates result extensions. Describes several other systems.

Source: Adapted from Machado et al. (2022).

Therefore, these skills are very relevant to understanding the level at which people are in terms of visual geometric thinking. Next, we move on to the methodological procedures of the research carried out in order to analyze whether its objective was achieved. That is, to analyze the visual geometric al thinking of participants in a didactic workshop when carrying out activities involving folds on a sheet of paper and identify geometric elements present in these folds.

3. Methodological procedures

This article addresses a mini-course/workshop, "Geometry as a didactic for teaching," offered by the researcher in September 2024 at a regional event in southern Brazil. This research is qualitative, as indicated by Moreira (2011): "Qualitative research is rooted in a paradigm according to which reality is socially constructed" (p. 42). In other words, in this article, we expected to explain how participants in a didactic workshop can develop visual geometric thinking through activities with folds on an A4 sheet involving fundamental elements of plane euclidean geometry.

Moreira (2011, p. 64) states: "[...] The special value of qualitative research —exploring meanings in real-world situations— must be preserved [...]." As this is a group interested in the topic and with pedagogical/mathematical training, the character denoted by the case study cannot be left aside since the author believes "the characteristics of a part are largely determined by the whole to which it belongs" (p. 86). Thus, when investigating the group in question, one can infer some difficulties that the researcher's experience of over fifty years thinks represent the community that teaches geometry at all levels of education.

The tasks included in this research involved easily acquired teaching material provided by the researcher. That is a sheet of A4 paper, pencils, and pens of different colors. The researcher proposed that participants complete the provided recording sheet with their data and answer nine items given under guidance. During the exercise, the researcher gave participants a short break so they could complete the worksheet and provide feedback. At the end, a tenth item was proposed, in which participants should indicate whether they liked geometry and, if possible, justify their answer.

The items included in the research are indicated in the analysis of the data collected so that they do not become repetitive throughout the text. The recording sheets were collected at the end of the task and analyzed later. In what follows, the data analysis is presented.

4. Data analysis.

The study counted eight individuals identified by their initials: Ad, Am, An, Di, Gi, He, Ma, and Ro. Six are taking postgraduate studies (master's or doctorate); one has a high school education, another a pedagogy degree, and the third has a PhD. Of the eight participants, three do not work as teachers. Of the others, two work in the initial years of elementary education, one in higher education, another works simultaneously in primary and secondary education, and the rest work in the final years of elementary school (middle school).

Therefore, all have already taken the geometry subject either as students or as teachers, which justifies their having basic knowledge of this area of knowledge. This research analyzes how those participants' visual thinking was constructed to perceive the relationship between the basic concepts of geometry and a didactic practice that can be used to construct such concepts.

The first question analyzed was: "Fold the sheet at any point A, creasing it well. If you prefer, reinforce it with a colored pen or pencil. Which geometric element represents the folding line on the sheet?"

Four respondents, i.e., half of them, stated that the element is a straight line, although they could identify it as a straight-line segment because the line was delimited by the A4 sheet provided. After the registers, the instructor discussed this with them and did the same in the subsequent activities. One of the individuals said it was a rectangle, which suggests that he did not pay close attention to what had been requested since the element was the fold line, creased and colored, denoting a well-defined visual aspect. As in the previous case, three individuals said it was a triangle, which makes no sense.

Apparently, even in this first activity, the visual aspects of half of the group seem not to have been well developed in the initial education, so they did not identify one of the first geometric elements —a straight line being distinguished from a curved line, for example, or from a straight line segment. This initial analysis leads to the non-visual level indicated by Anwar and D JuandI (2020) in Chart 1: students do not involve visual thinking in solving geometric problems, a non-visual characteristic at the indicated level.

The second question states: "Fold the sheet at A so that the right side overlaps the left and crease it well. If you prefer, mark it with another color. Which geometric object is marked in this second fold? How does it relate to the first geometric object?"

The same participant who had answered the first question (He) reiterates his answer that it is a rectangle. "It's the same shape, but the dimensions have changed." The two who had said it was a triangle in the first answer also reiterated their answer, with one of them seeing "symmetrical triangles" (Am). Two individuals said that they were two parallel lines. Participant An went a little further than requested by indicating that it was "Another straight line... An angle of 90º degrees between them." The notation is justified because he is in high school. Only two individuals said they were perpendicular lines, coherently, as the visualization of the objects represented by the fold lines was proposed. It is possible to notice difficulties at level 1, indicated in Chart 2, regarding the verbal skill "Associates the correct name with a given figure. Interprets sentences that describe figures" (Machado et al., 2022).

The third item requested in the construction of the folds was: "After the second fold, what happens in the first, i.e., which geometric objects are defined in the first?" The answer was expected to be "two opposite half-lines" or even "two line segments" if the limitation of the edge of the sheet is considered. The following registers were obtained:

He: Four rectangles with different dimensions.

Ad: Right triangles.

Gi: Triangles.

Ma and Ro: Parallel lines.

Na: Common point between two lines.

Am: Two perpendicular lines.

Di: Point and Line→ Half-lines.

There is much difficulty connecting previous and basic knowledge of plane geometry in the exploratory activity of folds and refolds on an A4 sheet, an easily accessible teaching resource for the teacher in his/her classroom. This lack of connection leads to a deficit in the development of visual thinking, that is, coordination between visual space and tactile-kinesthetic space, as pointed out by Piaget and Inhelder (1993).

Only one of the participants could identify two half-lines, which could have been added by the observation that they are opposite. The others did not see such objects.

Let us see what the representation would look like up to this stage (Figure 1):

Figure 1. Construction up to the third stage

Source: Prepared by the author

We can see that the fold lines do not form triangles or rectangles, and they may have analyzed the edges of the sheet that do not correspond to fold lines.

Moving on to the fourth proposition: "Choose a point on this second fold and call it B. Repeat by reflecting the second fold from B onto itself. If you prefer, mark it with the first color. What is the geometric relationship between the first and third folds?" Here, we present the participants' records.

> *He: Both are the same shape but with different dimensions.*
>
> *Ad: A square appeared.*
>
> *Di; An; Ma; Ro: Parallel [lines].*
>
> *Am; Gi: Perpendicular [lines].*

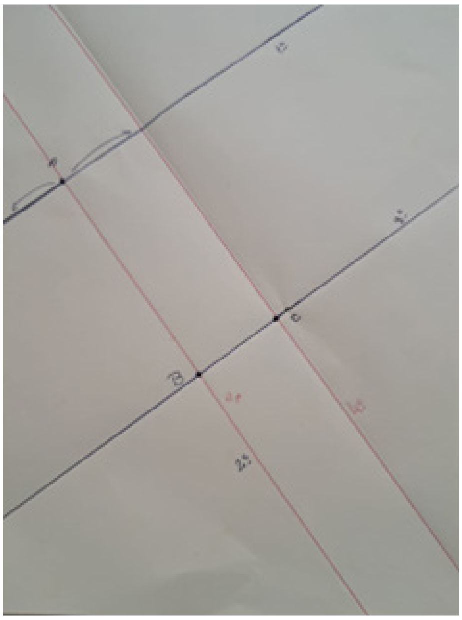
Half the class correctly visualized that the two marked lines were parallel (the 1st and the 3rd). However, it is worrying that the other half could not correctly identify their own construction, considering that seven participants are postgraduates or currently studying. Furthermore, five respondents work as teachers, which leads us to conclude that these individuals' visual thinking was poorly constructed, which will continue to leave gaps in their students' education. Thus, we anchor this above with Santiago and Ciríaco (2023) statement: "[...] spatial questions are brought up by children, intuitively, when they arrive at the early childhood education environment, and this should be the outset of the exploration of geometric thinking" (p. 6). Although concepts of plane geometry involved in the folds are explored, the object of exploration is concrete, while the geometric entities are abstract.

The fifth item of the didactic application consists of: "Now choose a point C on the third fold and repeat the previous procedures. If you prefer, color it the same color as the second fold. What is the relationship between the fourth and the first fold? And between the fourth and the third?" Their written answers are presented below:

> *He: Same as the previous answer.*
>
> *Ad: Two triangles appear.*
>
> *Di: Perpendicular lines, ditto.*
>
> *An: Just like the other examples, they are perpendicular.*
>
> *Ma: Parallel [lines].*
>
> *Ro: Perpendicular lines.*
>
> *Gi: Perpendicularity, parallelism.*
>
> *Am: [Left blank].*

Again, there are some cognitive conflicts in identifying relative positions between lines in the plane. He and Ad establish incorrect relationships, while Ma only identifies the parallelism between the 3rd and 1st, not the perpendicularism of the 4th with the 3rd. The same occurs with Ro; however, she does not identify whether perpendicularism is for the first or second question. Gi, on the other hand, is the only one who could answer both questions correctly. Figure 2 displays the colored lines in the foldings. We note here the local visual characteristic that students should already be able to distinguish, that is, the relationship between different images in geometry, in this case, the relative positions between lines, as indicated in Chart 1, as a visual-global characteristic indicated by Anwar and D JuandI (2020).

Figure 2. Construction of the fourth fold.



Source: Prepared by the author

The following proposition is: "The fourth fold meets the first at a point D. What is the geometric element obtained by lines AB, BC, CD, and DA?" The answers were:

In: Line segments.

Am; Di: Quadrilaterals.

Ad: A square.

Gi: A trapeze.

He; Ro; Ma: A rectangle.

From these, it becomes clear that Na and Gi provided inappropriate registers due to the lack of adequate nomenclature or because their folds did not follow the researcher's guidelines.

Ad obtained a square, which is perfectly possible since no distances were provided for point allocation. The last three have rectangles. We see that obtaining a square and a rectangle is correct because parallel and perpendicular lines were sequentially found. There is an advance in the development of visual geometric thinking in these individuals, when we explore Leivas's (2009) imaginative, intuitive, and creative skills.

The seventh question asked them to indicate: "And which is the geometric entity that these segments limit?" The results were as follows:

Ma and Ro: Parallel lines.

Ad: Geometric segments.

Gi: A trapeze.

Am: A quadrilateral.

He; Na; Di: A rectangle.

The last four answers characterized geometric figures, but what was requested corresponded to the geometric element "region," i.e., a quadrilateral or rectangular region. This differentiation between polygon and polygonal region corresponds to the "Geometry and Shapes" block, announced by BNCC (Brasil, 2018), which addresses lengths in the section on quantities and measurements, while the polygonal region will have, as its correspondent, the area. Therefore, it is vital to highlight the measurement aspects (numerical) from the geometric aspects (shapes), with the latter being better directed toward the visual.

The eighth item was: "Assign a position to segments AB and BC; AB and CD; BC and CD; BC and AD." The corresponding answers were:

> *He: A triangle, a rectangle, a triangle, and a rectangle.*
>
> *An: A perpendicular, parallels, a perpendicular, parallels.*
>
> *Di: Perpendiculars, parallels, perpendiculars, parallels.*
>
> *Am: Perpendiculars, parallels, perpendiculars, parallels.*
>
> *Di: Perpendiculars, parallels, perpendiculars, parallels.*
>
> *Ad: [Did not complete the answer].*
>
> *Ma: A perpendicular, parallels, a perpendicular, parallels.*
>
> *Ro: Perpendiculars, parallels, perpendiculars, parallels.*

It is possible to verify that only He did not answer in terms of relative positions between the marked segments. Ad left the question blank, and the others correctly described the positions of the segments that make up the figure. Here, we reach a more advanced level, as indicated by Van Hiele's theory: the analysis. In this case, the student "Perceives the properties of a figure as part of a larger figure," which, in this case, is the expected characterization of a rectangular figure based on the relationship between sides and angles. Furthermore, the answers indicate progress in developing students' visual geometric visual thinking, which can also be seen in the following item.

Question 9 requested: "Each pair of lines meet at a point forming angles. What is their measurement?"

He, like the previous item, left it blank. All the others indicated that they were right angles or 90º.

To conclude the investigative activity, they were asked to indicate the answer to the question: Do you like geometry? (   ) YES   (   ) NO (   ) KIND OF.

Everyone answered "YES," and only one respondent did not justify it. The other participants' justifications are recorded below.

> *An: It's fun content and has a lot of practical applications, especially with the technologies available.*
>
> *Di: It's the part of mathematics where we can build concepts with students.*
>
> *Am: Knowledge necessary to relate to objects in the environment and their measurements/representations.*

> *Gi: I like it. But, I don't quite understand.*
>
> *Ad: Yes, but because I have been away from the classroom for some time, I found revisiting the basic concepts worked on difficult.*
>
> *Ma: Well, it is an area of mathematics we can demonstrate to the students, and they can experiment with it.*
>
> *Ro: I like basic geometry because we follow our intuition to define the different shapes from there.*

This question shows us the participants' motivation for choosing the mini-course/workshop, even though they represent a small percentage of participants in the event, consolidating the perception of the abandonment of geometry in the school and academic environment.

5. Final considerations

This article addresses qualitative research that analyzes participants' visual geometric thinking in a didactic workshop when solving tasks involving folds on a sheet of paper and identifying geometric elements contained in them.

The workshop, which occurred at a public event in September 2024 in Rio Grande do Sul, had eight participants, most of whom work in primary and secondary education. One participant is an undergraduate student in mathematics, six are postgraduate students in mathematics teaching, and one has a PhD and works in postgraduate studies.

The activities took place in a single meeting lasting two hours and consisted of nine questions involving folds in an A4 sheet, where students should visualize elements of plane geometry. The first was a straight line or straight line segment due to the limitation of the paper sheet. Thus, the expectation was that respondents would identify this diversification of two basic concepts at once, which did not occur for many, who registered triangles that were not visualized by lines, as is the case with the latter (a triangle is a polygon formed by three consecutive straight line segments with no intersection except for the common ends so that the end of the first coincides with the third).

Subsequently, a reflection of this first fold (representation of the straight line) was suggested on a random point marked to obtain two visual representations of perpendicular straight lines and, consequently, two opposite half-straight lines and right angles between them. From there, by placing a second point on this second line, reproduce the process, obtaining a third line perpendicular to the second and parallel to the first. Repeating it, on the third line, a fourth one is found, which, perpendicular to the previous one, becomes parallel to the second and meets the first at a fourth point.

Using the same color in the first and third and a second color in the second and fourth allows for a better visual aspect and facilitates the relationships of parallelism and perpendicularism. In addition, right angles are obtained between two consecutive folds. Thus, four points are obtained by delimiting four segments, two by two congruent or, even, all four, forming two by two right angles. Two important plane geometry elements that are not properly explained in textbooks result from that: a quadrilateral and a polygonal region. These elements define the quadrilateral and paint the region delimited by it, which does not correspond to a formation of visual geometric thinking.

We must consider that the polygon corresponds to a linear quantity (measurement of lengths and perimeter) while the polygonal region corresponds to a bilinear quantity (measurement of area). No participants visualized the quadrilateral region, only rectangles, and a square, depending on the random points chosen.

The research showed that the group has limitations in visual geometric thinking based on findings from using relatively simple but objective manipulative teaching material. We hope that new achievements from other groups can bring new contributions to the theme.

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