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Teaching knowledge and partial progression in mathematics: what happens when mathematics teachers are invited to reflect on the thematic unit Numbers?

Saberes docentes e Progressão Parcial: o que acontece quando professores(as) de Matemática são convidados(as) a refletir sobre a unidade Números?

Saberes del docente y progresión parcial: ¿qué pasa cuando profesores de matemáticas son invitados a reflexionar acerca de la unidad Números?

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Abstract
This article analyzes teaching knowledge mobilized by six Ouro Preto teachers while discussing and reflecting on the thematic unit Numbers in partial progression in mathematics. Based on a qualitative approach, it was developed with data from observations, a field diary, transcriptions of their recordings, and participants' registers. Data were interpreted in the light of the notion of teaching knowledge developed by Maurice Tardif. The results show that the unit is the most highlighted by the group. Based on experiential knowledge, they propose that the partial progression be developed from a survey that guides the elaboration of activities and culminates in three evaluations. The research concluded that besides experiential knowledge, curriculum knowledge and knowledge of the subjects also supported the group's argumentation and decision-making.
Keywords: Mathematics Education. Partial Progression in Mathematics. Teaching Knowledge. Numbers. Elementary School.

Resumo
Este artigo analisa saberes docentes mobilizados por seis professores(as) de Ouro Preto enquanto discutem e refletem sobre a unidade Números na progressão parcial em matemática. Desenvolvido em uma abordagem qualitativa, o estudo pauta-se em dados produzidos a partir de observações de encontros, diário de campo, transcrições das gravações e registros feitos pelos(as) participantes. Os dados foram interpretados à luz da noção de saberes docentes desenvolvida por Maurice Tardif. Os resultados evidenciam que essa unidade é a mais destacada pelo grupo e, com base em saberes experienciais, propõem que a progressão parcial seja desenvolvida a partir de uma sondagem que oriente a elaboração de atividades e culmine em três avaliações. Além dos referidos saberes, desvelaram-se os curriculares e disciplinares, que sustentaram tanto a argumentação quanto a tomada de decisões pelo grupo.
Palavras-chave: Educação Matemática. Progressão Parcial em Matemática. Saberes Docentes. Números. Ensino Fundamental.

Resumen
Este artículo analiza los saberes docentes movilizados por seis profesores de Ouro Preto mientras reflexionan sobre la unidad Números en la Progresión Parcial en Matemáticas. Desarrollado con un enfoque cualitativo, se basó en datos producidos a partir de observaciones, diario de campo, transcripciones de grabaciones y registros producidos por los participantes que fueron interpretados a la luz de la noción de Saberes del docente desarrollada por Tardif. Los resultados muestran que esta unidad es la más destacada por el grupo que con base en su conocimiento experiencial propone que se desarrolle la Progresión Parcial a partir de una encuesta inicial de conocimientos previos y tres evaluaciones. Además del conocimiento experiencial, se revelaron conocimientos curriculares y disciplinares que sustentaron tanto la argumentación como la toma de decisiones por parte del grupo.
Palabras clave: Educación Matemática. Progresión Parcial en Matemáticas. Saberes del Docente. Números. Enseñanza Fundamental.

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1. Introduction

In 2018, in my first experience as a teacher, even before completing my teaching degree in mathematics, I took charge of a seventh-grade class at a public school in Ouro Preto. I faced several challenges, including partial progression (PP) development. Students usually completed two assessment activities: an assignment (40 points) and a test (60 points). Approval would be subjected to a grade equal to or greater than 60 points (60%). Those who did not achieve this grade could attempt again the following semester. However, there was no guidance on developing the PP, nor were extra classes available to guide and prepare students for studying the content. I quickly realized that this used to be the norm in the region.

I wondered how students could overcome their difficulties with mathematical content just by doing a project and a test practically on their own. Such questions only deepened over time. From my perspective, Gomes's (2004, p. 47) words, said almost 20 years ago, are still relevant: with the PPM,

> (...) the flow of students is regularized and costs are reduced, satisfying managers and the eternal budgetary limitations of education; unwary parents and students are satisfied because, with no failure, there is an illusion of learning [...] [Therefore,] there is a risk of advancing in quantity and circumventing the problems of quality and democratization. And as students do not learn much, starting with the poorest, unfair structures are maintained.

PP is a public policy aimed, at least theoretically, at reducing repetition and dropout rates, as well as age-grade distortions. It represents the current version of a strategy established in the 1970s (Art. 15 of Law 5692, of August 11, 1971) under the name of "dependence," which assumed that, from the seventh grade onwards, the student could be "enrolled, dependent on one or two subjects, areas of study, or activity from a previous grade, as long as the sequence of the curriculum is preserved" (BRASIL, 1971).

Two decades later, this action was extended to all grades and became known as regular progression by grade (Law 9394/96, Article 24, Section III). In the specific case of Minas Gerais, Resolution SEE/MG 4692 of December 2021 is in force, establishing:

> Art. 105–Partial progression is the procedure that allows students to advance in their academic trajectory, providing them with new study opportunities in the subsequent academic year in those aspects of the curriculum components in which they still need to consolidate basic knowledge and skills.
>
> § 1–Partial progression is provided for from the 6th to the 9th grades of elementary school and in the 1st and 2nd grades of high school.
>
> § 2–The provisions of the caput also apply to the transition from the 9th grade of elementary school to the 1st grade of high school.
>
> Art. 106–Students may benefit from partial progression in up to 3 (three) curriculum components in the subsequent academic grade.
>
> Sole paragraph. Students promoted in partial progression are guaranteed enrollment in the subsequent school grade only in schools in the Minas Gerais state public education network.
>
> Art. 107–Students in partial progression must be assured of guided studies according to the pedagogical intervention plan prepared jointly by the teachers of the curriculum component(s) of the previous and the current grades to overcome gaps and difficulties in the

> object(s) of knowledge, skill(s) identified by the teacher and discussed in the class council (MINAS GERAIS, 2021).

Although this Resolution represents, "in its essence, the principle of respect for the rights of citizens and their individuality regarding cognitive development and the construction of knowledge, taking into account the time and pace of learning of each individual" (PESSÔA, 2016, p. 49), in many cases, it does not achieve its central goals in student learning; instead, it highlights the gap between the educational level of the most favored social classes and that of the least favored ones. I understand that teachers' systematic exclusion from decision-making processes related to carrying out PP influences this scenario. They are the ones who know the students best and interact with them on a daily basis; therefore, they would have more elements to outline the development of this public policy within the schools in which they work.

In short, PP is a national public policy created in the 1990s to reduce failure, age-grade distortion, and dropout without neglecting the quality of learning. However, in practice, the results in terms of learning do not correspond to expectations. In this scenario, it is striking that teachers, protagonists of the process together with students, are not invited to participate in decisions related to the development of this public policy.

Based on this problem, understanding the importance of recovering the teaching protagonism in the definition and implementation of the partial progression in mathematics (PPM), I proposed to investigate teaching knowledge mobilized when mathematics teachers from the public education network of Ouro Preto, Minas Gerais (MG) discuss and reflect on that policy[3] (FIGUEIREDO, 2023). Mathematics teachers' teaching knowledge (TARDIF, 2014) mobilized in discussions about PPM is the object of study of this research. I sought to develop it by carefully listening to six mathematics teachers from the Ouro Preto public schools, who voluntarily agreed to participate in a study group that held five remote meetings in 2022.

I present here an excerpt from this research. I analyze teaching knowledge mobilized by the group when discussing how the thematic unit Numbers could be organized in the PPM. To this end, I begin by briefly situating the notion of teaching knowledge from Tardif's (2014) perspective and then describe the methodology adopted, moving on to the results and analysis and closing with some considerations about the process.

2. Teaching knowledge

Even though limited by the pressures and constraints imposed by legislation and schools alike, teachers mobilize diverse knowledge/knowings when developing PP. Like Tardif (2014, p. 286), I understand that as professionals, teachers are "reflexive or 'reflective' practitioners who produce knowledge specific to their own work and are capable of deliberating on their own practices, of objectifying and sharing them, improving them, and introducing innovations likely to increase their effectiveness." Their knowledge "is plural and also temporal since, as stated, it is acquired in the context of a life story and a professional career" (TARDIF, 2014, p. 19-20) and "must be understood in close relation to their work at school and in the classroom." (TARDIF, 2014, p. 16-17). It "does not come from a single source, but from several sources and different moments in life history and pro-

3 Henceforth PPM.

fessional career [...]. Work experience, therefore, is just a space where the teacher applies knowledge, *being itself the knowing of work on knowledge*" (TARDIF, 2014, p. 21, author's emphasis), and the "idea of interactive work, that is, work where the worker relates to his/her work object fundamentally through human interaction." (TARDIF, 2014, p. 22).

The author defines knowledge as "encompassing knowledge, competencies, skills (or aptitudes) and teachers' attitudes, that is, what has often been called knowing, know-how-to-do, and know-how-to-be" (TARDIF, 2014, p. 60). To identify and classify teaching knowings, Tardif (2014, p. 62-63) proposes a typology that "attempts to account for the pluralism of professional knowing, relating it to the places where teachers work, to the organizations that educate them, and/or where they work, to their working instruments and, finally, to their working experience." This model seeks to contemplate teachers' knowings in their classroom practice. It is also possible to note that much of it is not produced directly by teachers and that some of the teachers' knowings, according to Célia Nunes (2004, p. 18), "are, in a certain way, external to the teaching profession, coming from places prior to the career or outside the work itself."

Chart 1: Teachers' teaching knowings

Teachers' knowing	Social sources of acquisition	Modes of integration in teaching work
Teachers' knowing	Family, living environment, education in the lato sensu, etc.	Through life history and primary socialization
Knowing from previous school education	Primary and secondary school, non-specialized pre-secondary studies, etc.	For pre-professional education and socialization
Knowledge from professional education for teaching	Teacher education establishments, practicums, refresher courses, etc.	For professional education and socialization in teacher education institutions
Knowing from programs and textbooks used at work	The use of teachers' "tools": programs, textbooks, exercise books, worksheets, etc.	By using working "tools", adapting them to tasks
Knowing from own experience in the profession, in the classroom, and at school	Craft practice at school, in the classroom, peers' experience, etc.	Through working practice and professional socialization

Source: Tardif (2014, p. 63).

This model highlights the social nature of professional knowing since "several of them are somehow 'external' to the teaching profession" (TARDIF, 2014, p. 64), and all of them are actually used regularly by teachers in their teaching practice. "In this sense, professional knowing is somehow at the confluence of various sources of knowing coming from one's life history, society, the school institution, other educational actors, places of education, etc." (TARDIF, 2014, p. 64). Célia Nunes (2004, p. 31) describes this knowing as follows:

> The "personal" ones, acquired in life and education in the broad sense and integrated by life history and primary socialization; the "school education" ones, originating from schooling and integrated by pre-professional education and socialization; the "education for teaching" ones, originating from professional education courses and integrated by education and socialization in education institutions; the "programs and textbooks" ones, used by teachers as "tools" and integrated into teaching work, and the "experience" ones, acquired in the practice of the school profession, with students, peers, and integrated into work through professional socialization.

In short, teaching knowing is all that is acquired and mobilized by the teacher in terms of work. From this perspective,

> Everything is knowledge: habits, emotions, intuition, ways of doing (the famous know-how-to-do), ways of being (the equally famous know-how-to-be), opinions, people's personalities, ideologies, common sense, all rules and norms, any everyday representation. [...] In our view, the problem does not consist in affirming the existence of informal, everyday, experiential, tacit knowledge, etc., but in designating these different types of knowledge through an imprecise, undefined notion (TARDIF, 2014, p. 192).

The notions briefly presented here underpinned the research development from which I extracted the excerpt made for this article. They provided the theoretical lenses for analyzing the discussions and productions of the group studied. When I met with colleagues, mathematics teachers, to reflect on PP in this discipline, I created a space in which teaching knowledge emerged. As mentioned previously, this article focuses on the teaching knowledge mobilized by the group when discussing how the thematic unit Numbers could be organized in the PPM.

3. Methodology

The nature of the phenomenon under study –teaching knowings mobilized in a study group– demands a qualitative approach since such knowings are "a social reality materialized through education, programs, collective practices, school subjects, institutionalized pedagogy, etc., and is also at the same time, *its knowings*." (TARDIF, 2014, p. 16, emphasis added). In this sense, the research developed could be called exploratory.

The study was developed within the scope of the public schools of Ouro Preto. In total, 41 public schools provide basic education in the municipality. After approval by the Research Ethics Committee[4], an electronic form was forwarded to 58 mathematics[5] teachers from the schools mentioned above. The form aimed to identify understandings about the PP and invite the teachers to participate in the study group. Thirty-three teachers answered the form, of which 14 said they were interested in participating in the research. Given the difficulties in reconciling days and times, the group was formed by six teachers and the researcher, who also teaches mathematics in the public education system in Ouro Preto. Except for the researcher, each teacher is briefly introduced below with pseudonyms.

João has a teaching degree in mathematics, over 30 years of experience in 2022, and, at the time, was a permanent teacher at a state and a municipal school. He was deputy secretary of education and principal of two schools; during the pandemic, he worked as coordinator of the mathematics area at the Municipal Department of Education.

Carla has a teaching and a regular degree in mathematics, and at the time of the study, she was teaching in three schools (two municipal and one state) as a contractor. She had over 15 years of experience. He said he would have completed the pedagogy course by the end of 2022 and had taken some postgraduate courses.

4 CAAE: 51547921.8.0000.5150.

5 Given the difficulties in obtaining information from the Municipal Department of Education and the Superintendence of Education, I visited schools in Ouro Preto and consulted colleagues about mathematics teachers. For more details, see Figueiredo (2023).

Helena had a teaching degree in mathematics and a professional master's degree in mathematics education. She had over 12 years of teaching experience.

Joana held a teaching degree in mathematics and a specialization in mathematics teaching methodology. She had over five years of experience and worked at a school in the Ouro Preto municipal education system as a contractor in 2022.

Taís has a degree in mathematics, a specialization in mathematics education, and a professional master's degree in mathematics education. At the time of the research, she had over eight years of teaching experience and worked as a contract teacher in the municipal education system of Ouro Preto.

Heloísa has a teaching degree in mathematics. She taught for over 30 years in the state education system. At the time of the study, she had been working for over 20 years as a contract teacher in the municipal school network of Ouro Preto.

The data were produced from observations of the meetings registered in the researcher's field diary, transcriptions of recordings of the meetings, and participants' registers. Data analysis was developed through the interpretation of data in light of the notion of teaching knowings developed by Tardif.

4. Group reflections on the thematic unit Numbers

I invited the interviewees to express *what they thought about the PPM* to which João, the most experienced teacher in the group, answered:

> [...] every opportunity is very valid, even more so in our mathematics subject, where we know that students have a lot of difficulty. It's a subject that historically has a high failure rate, although today, the change and the way of assessing... yeah... failure is avoided (L175 to L181).
>
> We had some experiences in the state about the progression that, at first, was called dependence [Note of the translator: When the student retakes the failed subject without having to repeat the whole grade], then, they called it progression.[...] This is the model that is in place now, it is a model that makes us very concerned, we don't see any progress, right, the name is progression, but there is actually no progress, it is something that is done just to comply, comply with the norm, right, and that really worries us a lot (L07 to L14).

The literature corroborates the teacher's speech. Cláudia Nunes (2008, p. 74) highlights the "distance between the creation of public inclusion policies and their implementation" in schools. However, João states: "I am not against progression. I am in favor as far as opportunity is concerned" (L183 to L186). For the teacher, it is necessary to think of the students and provide the support they need. For Carla, the PPM is treated as a bureaucratic task in schools, and how it is carried out suggests that those involved do not fully understand it. Taís reinforces this idea by stating: "...I think that progression, the way it is working, it doesn't actually work, because it puts the student in a partial progression and nothing happens to that student" (L146 to L150).

Taís, probably based on her experiential knowing, assesses that the PPM has been transformed into an additional "burden" for teachers; however, she says that students do not have support to do the tasks requested in the process:

> I think it overloads the teacher because they are there thinking about their lesson plan for that year and having to assign work to the student... (L163 to L171).
>
> If there are no conditions, if there are no classes, if there is no way for the student to learn what they did not learn during the year, it is not an efficient progression. They will just do extra work the following year. So, I think the progression, how it is done, makes no sense (L152 to L156).

The teacher's statements refer to Torres' ideas (2001 apud LIBÂNEO, 2012, p. 18):

> The expanded vision of education has shrunk, i.e., a) of education for all, for the education of the poorest; b) of basic needs, for minimum needs; c) of attention to learning, for the improvement and assessment of the results of school performance; d) of improving learning conditions, for the improvement of the internal conditions of the school institution.

Almeida (2012, p. 58) states that the "partial progression policy did not positively influence the recovery of learning, thus failing to achieve its main function: to provide students with the opportunity to continue their school trajectory without interruption." Carla seems to think similarly because, according to her,

> The way it has been happening is not giving opportunities; on the contrary, it is taking away the opportunity. Now, if you do the work well, in the opposite shift, with the teacher covering all the content of that grade and the student attending class as it should be, then yes, it is giving an opportunity. But as it happens today, no, I would say that ***it is not even "a draw"; it is taking away the opportunity*** (L515 to L521, my emphasis).

In short, for the teachers participating in the study group, PP is a public policy far removed from the reality of the classroom, requiring significant changes to fulfill its role. For them, the current format does not create a real opportunity for students to recover learning that has not been consolidated but rather a false illusion. Furthermore, they note that students and also teachers have trivialized the PPM. However, they do not rule out that this public policy could be valuable if it is well-developed.

Another highlighted dimension is the teacher's lack of autonomy in conducting the PPM. They feel dependent on the pace and directions taken by the school management and pedagogues. In some situations, such referrals even involve guidance to avoid student retention, regardless of their performance and commitment.

During the meetings, the group began to discuss how to reorganize the PPM. Questions about the time available to work on specific content, the excessive volume of content, and the emphasis given to some thematic units (TUs) to the detriment of others were raised. The group concluded that just one assignment and one test (current model) would not be enough to cover important content and suggested adopting priority levels for the content. After reflections and discussions, the idea of organizing the PPM curriculum into three stages, developed throughout the year, came to the fore. In the first, topics from the TU Numbers would be covered; in the second, TU Algebra would be prioritized; and in the third, TU Geometry. The other two TUs (Quantities and Measurements and Statistics and Probability) would be addressed concomitantly, seeking to correlate the contents that favor their understanding. The stages could be subdivided into assessment tasks and three assessments during the school year. Furthermore, it became clear to the group the importance of

developing and applying, before starting the tasks, a survey of mathematical knowledge, which the participants called a "diagnostic assessment." The instrument analysis would offer a reference for the selection of mathematical topics to be addressed in each TU.

5. A proposal to address the TU Numbers in the PPM

From the first meetings, it is clear that the group is particularly worried about the TU Numbers. The other units are not so evident; however, this does not mean they are disregarded.

> All operations with natural numbers, then with decimals in rational form, end up being a large volume of things, right? (Helena, 3rd meeting, L178 to L183)
>
> It seems like a very simple and obvious thing, but when we start working with them, they still have this doubt and many of them even reach high school, it's... If they don't have any doubts, they may even get to high school without knowing what it is. So, for me, the part about the set of natural numbers, talking about what these natural sets are, what they are..., that's the kickoff. (Carla, 3rd meeting, L71 to L75)
>
> If he/she doesn't know how to work with this, it will stop them when they get to geometry. This will stop them when they face a statistical probability. (Carla, 3rd meeting, L210 to L214)

João agrees with his colleagues and argues:

> If it were in my view for the sixth grade, I would also only stay in the thematic unit for the sixth grade with numbers, I would give up the statistical probability part, I would give up geometry. And I would work in the sixth grade just with operations, both natural numbers, fractions, and decimals, I think that if we work in the sixth grade, which is more important, it is... Stay in the thematic unit Numbers and operations, right? (3rd meeting, L113 to L125)

When selecting the content for the seventh-grade PPM, Helena says she faced "difficulties" "because it is too extensive" for the "time available." When referring to the content that should be covered in those students' PPM assessments, she argues:

> So, yeah, I decided to take a look at the BNCC and the textbooks, and then, based on this, on this principle that [...] is that I think it is partial progression, it must be the minimum, the minimum necessary, because there's no time to go through everything the document proposes. It's within the Numbers axis, yes, I think we should work with whole numbers. (3rd meeting, L311 to L328)

Like Tardif and Lessard (2014, p. 211), I observe that:

> Teaching is, therefore, constantly making choices in full interaction with students. Now, these choices depend on the teachers' experience, knowledge, convictions and beliefs, commitment to what they do, representations regarding the students, and, evidently, the students themselves. (Tardif, 2014, 132)

In this context, when analyzing the curricula and thinking about the PPM, the participants prioritized the Numbers TU in all grades, given its importance in students' education, as they considered it a prerequisite for developing most skills. The TU Geometry and the TU Algebra were considered from the seventh grade onwards.

Based on the discussions, the following summary table was organized for the TU Numbers:

Chart 2: Content of the 1st assessment on the 6th-grade content

Planning for Partial Progression		
Object of knowledge	**Skills**	**Justification for Inclusion**
- Decimal numbering system: characteristics, reading, writing, and comparison of natural numbers; - Use of the number line.	(EF06MA01A) (EF06MA35MG)	- Fundamental ideas about natural numbers; - Recognize the main aspects of natural numbers: reading, writing, and comparison;
- Operations with Natural Numbers IN: Addition and subtraction of natural numbers; Multiplication and division of natural numbers; Power of natural numbers; Square root of perfect squares.	(EF06MA36MG) (EF06MA03)	- Fundamental ideas about operations that will focus on operations on whole and rational numbers.
- Prime and composite numbers. - Multiples and divisors of a natural number.	(EF06MA05) (EF06MA06)	

Source: Figueiredo (2023, p. 147).

The excerpts from the dialogues that took place at this meeting, presented in Chart 3, illustrate how the group reached this synthesis:

> Ricardo: In the sixth grade, I thought the following [...] the four operations are essential. [...] The idea of divisibility is very important, right? [..] which would help in the study of rational numbers. What do you think [...] about the idea of divisibility being a basis for students to work with rational numbers? (L48 to L56)
>
> Carla: If you are working with progression, you also have to assume that this student already has some experience. [...] This [divisibility], in my opinion, is a content that you can leave to the students themselves [...] Yes, other things are more important [...]. Mainly prime numbers, it's something that students must... understand. (L57 to L102)
>
> Helena: What about fraction isn't it missing here? Wouldn't it go under Numbers? (L133) [...] According to the guiding document, I entered rational numbers, the part of the item that is the knowledge objective, is related to fractions. (L137 to L153)
>
> Carla: I agree, I agree... Fractions, I work with it, but I'm in regular [classes], it's not a progression, but even so, in a progression, why would I put fractions? (L154 to L163)
>
> João: Actually, if we adapt this new Minas program, we will really have to see more fractions in the seventh grade because there is an active program there, right? And we end up... following a little bit, [because] we worked a lot on fractions in the sixth [grade], but if those kids are having a hard time, now it's time for us to actually review it, right? (L182 to L207)

.The teachers' statements give evidence[6] of experiential knowledge. An example can be seen in Carla's advocacy of Multiples and Divisors. Another example is when Helena emphasizes the importance of not leaving out fractions for the sixth-grade PPM, highlighting that the curriculum guides the continuation of the study of fractions from the initial years of elementary school and that it is important to work with rational numbers, with their various forms of representation of rational numbers (fractions, decimals, and percentages) in the seventh grade. Furthermore, Carla highlights that fractions should be worked with more concrete examples, with a differentiated approach, since students generally find it very difficult to understand the topic. These are just a few instances of this knowledge.

6 The term "clue" is used to refer to data that expresses a lead, a clue. In this case, the selected statements are data produced from the transcripts of the meetings and the field diary that provide clues to experiential knowledge.

For the discussion of the seventh-grade PPM, I present Chart 3, prepared by the group as a summary of the discussions:

Chart 3: Contents of the 1st assessment on the 7th-grade content

Planning for Partial Progression		
Object of knowledge	**Skills**	**Justification for Inclusion**
- Rational numbers in fractional and decimal representation: uses, ordering, and association with points of the numerical line and operations.	(EF07MA10) (EF07MA11) (EF07MA12)	- Application in various everyday situations; - Knowledge necessary to understand other mathematical content; - Content that students generally have difficulty with.
- Whole numbers: uses, history, ordering, association with points on the number line, and operations.	(EF07MA40MG) (EF07MA03) (EF07MA41MG) (EF07MA04)	- Whole numbers are present in many everyday situations; - It is necessary knowledge for learning other mathematical content.

Source: Figueiredo (2023, p. 149).

This chart production originated from intense discussion. Here are some excerpts to illustrate:

> Helena: I put rational numbers, fractional and decimal representation. [...] And that's where [...] this operational part comes in, [...] the student must have this knowledge already. Otherwise, it will be too long because I can't leave whole numbers aside either, you know? (L414 to L423)
>
> Carla: Basically, we would do the following: in the sixth grade, we would work on the fractions of positive numbers, and in the seventh grade, on negative numbers. (L424 to L426)
>
> Helena: In the sixth grade, this question of calling it a rational number doesn't appear, right? Only this fractional representation part appears, but the idea is practically the same. (L427 to L429)
>
> Ricardo: I think that in the seventh grade, it begins by focusing more on operations and manipulations, on transforming decimals from fractions to decimals. (L432 to L438)
>
> Helena: So that's it, it's because there's a huge part of the document where it says fractions and their meanings as part of whole numbers, result of division, ratio, and operation. This is where the issue of comparing fractions comes in, the result of division is the idea of part-whole, etc. Then the operations come into play, but then comes the other axis within the numbers, which is like rational numbers. (L439 to L446)
>
> John: I would do the whole numbers first. Then, we would work on the rationals with the operations that we must synthesize. So, I would do the whole numbers, I think it takes a lot of time for them to understand signs well, and then work on the rational numbers too, the positive and negative rational numbers. I would change the order there (L452 to L455)

Participants discuss the separation and ordering of rational numbers and whole numbers in the application of the PPM. Their concern with learning fractions is corroborated by Patrono (2011, p. 23), who, in her research, found results that "showed difficulties in all the proposed questions (representation, comparison, equivalence, operations and application of fractions in problem situations) and all grades." For the author, one of the difficulties would be understanding that the rules established for natural numbers now need to be rethought for rational numbers. In this sense, Helena relies on her knowledge of the curriculum to highlight that she considers it necessary for operations to be worked on in the seventh-grade PPM, as the student would have already expe-

rienced their representations in the previous grade, which would contribute to learning the topic. However, João bases his knowledge on experience, according to which whole numbers should be explored before operations with rational numbers, given students' difficulties in understanding the rules of signs.

Regarding the eighth-grade PPM, I present Chart 6, prepared by the group:

Chart 4: Content of the 1st Assessment on the 8th-grade content

Planning for Partial Progression		
Object of knowledge	**Skills**	**Justification for Inclusion**
- Rational Numbers. - Numerical Sets (Natural, Whole, Rational, Irrational, and Real Numbers).	(EF08MA28MG)	- Content enables understanding and expanding knowledge of rational numbers in decimal and fractional form; - It allows us to identify regularity patterns; - Generating fractions will be content that helps us understand irrational numbers.
- Repeating decimal and generating fraction	(EF08MA05)	
- Percentage.	(EF08MA04)	

Source: Figueiredo (2023, p. 152).

Excerpts from the discussion held on the PPM in the eighth grade illustrate these ideas:

> João: Then, in the ninth grade, he also starts with numbers, right? [...] But let's make it smaller. Let's keep numbers to a minimum and strengthen algebra. (L775 to L782)
>
> Ricardo: Yes, I just put repeating decimal here because what I thought when I put repeating decimal is that it has elements to understand what an irrational number is, for example. (L784 to L789)
>
> Carla: But just look at the percentage, students, they... Even though it's the eighth grade, right, it's something that you use a lot in your daily life, right? [...]. If there is time, in the end, then I would approach it, yes. You work with financial mathematics in it, it's not a question of interest, discounts, additions, all that, you work on the percentage. (L792 to L809)

The speeches signal the mobilization of experiential knowledge. In this context, interaction plays an important role in participants' understanding, as they agree with the idea that the area coordinator will contribute to the PPM's work being developed more appropriately for each context, seeking other sources of knowledge to guide and direct teachers' work in the education network. Tardif (2014) reinforces that collaborative work between teachers is part of sharing knowledge that occurs naturally and without obligation or need.

For Célia Nunes (2004, p. 93), socialization among teachers is a condition for the production of knowledge related to experience, which acts "in a tutoring manner for those who recognize themselves as possessors of teaching knowledge, [so] that the teacher finds the possibility to go beyond what he/she knows, thus constructing new knowledge." In my understanding, this climate of collaboration and interaction between teachers provides and mobilizes knowledge. In this sense, João and Helena present their final words about the meetings that took place in this work:

> João: The guidelines, what we are going to teach, a plan built together. I think this is already a gain, the way of working (L511 to L520). [...] It is not a closed document; when a document is

> not closed, it may be improved, right, we can grow together as we apply it, always re-evaluating it, right? Because that is the path or a continuing action. (L708 to 712)
>
> Helena: That is what I think: everything that has been done so far has been a very down-to-earth proposal, you know? Our experience in class contributed. The guiding documents also helped us define the minimum necessary. (L521 to L527). [...] And yes, it ends, it was your anxiety, but it was everyone's, because something we saw wasn't working. [...] So here, right, everyone, with their experience, could contribute to building something that is possible, so that's how it is. (L717 to L730)

In short, participants mobilized experiential knowledge, drawing extensively on their experiences in the classroom to formulate strategies and teaching methods that could support students in overcoming their PPM deficit. The knowledge of the subjects highlighted in this meeting occurred in the context of school organizations regarding schedules and assessments resulting from the regular school year. They also propose periodic interventions in the classroom, taught regularly, to focus on the content to help students recover in PPM. Another suggested action is extracurricular activities (homework) to check students' development in PPM. This only corroborates Célia Nunes' (2004) ideas when she highlights that socialization is a source of knowledge production.

6. By way of summary

Teaching is an activity focused on teaching and learning and occurs in the interaction between teachers and students in the school environment. In turn, this environment is an organized and social space that influences the actors' work. To perform their role, teachers produce and mobilize diverse knowledge. In this context, the development of public PP policy, although aimed at creating new learning opportunities, involves organizational issues and has generally been developed in schools without teachers actively participating in the process and decision-making, which makes teaching work difficult and, consequently, affects the results achieved.

In this article, I sought to reveal knowledge mobilized by mathematics teachers when discussing and reflecting on the approach of the thematic unit (TU) Numbers in partial progression (PP). Thus, I effectively focused on the data from five meetings with a group of teachers. During the meetings, the TU Numbers was the most highlighted. The group agreed that this unit would be the kickoff for learning the other TUs. Thus, considering the extension of the mathematics programs and the limited time for developing the PPM, there was a consensus that this TU should be the priority.

During the meetings, study participants were invited to discuss and reflect on PPM, focusing on improving learning in the aforementioned process. Initially, there were several criticisms about how this public policy has been applied in schools, highlighting the lack of time and support necessary for its development, both for teachers and students. For the group, the PPM does not fulfill its role, as it does not adequately allow for the recovery of learning that has not yet been consolidated. However, teachers recognize its importance as an opportunity for students to appropriate content that has not yet been consolidated in the previous grade if it is improved and well conducted.

By mobilizing knowledge (mainly experiential, but not only), participants begin to outline new possibilities for developing PPM. They conclude that a single assessment is neither sufficient

nor adequate to cover the content considered essential for quality learning and discuss a PPM proposal structured in three stages.

Thus, based on their knowledge and the collective analysis of the curricula from the sixth to the eighth grades, they define an order of priority for the contents to be considered in the PPM. For the sixth grade, discussions revolved around the need to consider introducing the ideas of rational numbers and not just natural numbers as occurred; however, due to the priority considered, only natural numbers were considered. This option was based on the argument that seventh-grade students could consolidate any existing gap. In this scenario, the teacher should seek to observe these students in PPM and support them in this content.

For the seventh-grade PPM, teachers consider rational and whole numbers. In this assessment, two discussions took place: the first involved which of the sets should be approached first, and the second questioned how rational numbers should be approached since, in the sixth grade, these are seen as "fractions and their meanings as part of a whole," and in the seventh grade, as rational numbers. The teachers relied on knowledge of the subject contents to present their arguments. When mentioning that students have some difficulty understanding both sets of numbers, in the case of whole numbers, the difficulty would be located mainly in the "sign rules." When pointing out knowledge about students' difficulties, teachers relied on experiential knowledge. For the eighth grade, the importance of percentages and the importance of TU algebra are mentioned in arguments related to experiential knowledge. However, the Numbers unit prevails in the evaluations.

The results presented in this section show that teaching knowledge emerges and is enhanced by collective reflection. In just five remote meetings, the participants in this study showed that when teachers are invited to collaborate, they can go beyond criticism and propose concrete actions that could improve the application of this public policy.

The meetings promoted were divided into two moments. In the first moment, the teachers complained against and criticized the PPM, highlighting discontent and the need for change in the way this policy is applied in schools in the Ouro Preto public education network. Then, from the second meeting onwards, teachers began mobilizing knowledge to precisely reverse the scenario through their interactions and discursive exchanges, which focused more on solving the problem.

Experiential knowledge stood out as the most important knowledge mobilized in each meeting. In my view, it is natural for this to occur, given its very constitution as knowledge, since, in their speeches and arguments, teachers often base themselves on everyday teaching experiences and their reflections on them. It is also necessary to consider that initial education often does not significantly promote the construction of mathematical knowledge specific to teaching, privileging academic knowledge that does not dialogue with the demands of teaching practice. Therefore, such knowledge (from education) is little mobilized when dealing with such demands. Furthermore, in everyday practice, experiential knowledge produces a certain objectivity regarding other kinds of knowledge; therefore, the knowledge teachers acquire during their education (initial or continuing) or from curricula and school programs undergoes a retranslation as a result of this evaluation of other kinds of knowledge. In this way, teachers eliminate what seems abstract or has no relation to reality, keeping what they think to be valuable (TARDIF, 2014).

Although less frequently, other types of knowledge were also mobilized. The aspects of curriculum and subjects, for example, supported the arguments presented at various times. Although occupying a position of exteriority concerning teachers, these types of knowledge often merge with experiential knowledge and are reinterpreted. It is also important to highlight that teaching knowledge rarely appears in isolation. Teaching knowledge is complex and plural, and identifying it constitutes more of a scientific issue for analysis and understanding of the phenomenon than a teaching practice demand.

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