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Articles Section

# Etic (Global) Ethnomodels based on the Bottle-Cap Spinning Top: A Contribution to Integral Calculus from Ethnomodelling

## Etnomodelos Éticos (Globales) basados en el Trompo de Tapitas: aporte al Cálculo Integral desde la Etnomodelación

## Etnomodelos Éticos (Globais) baseados no Pião de Tampinhas: Contribuições ao Cálculo Integral a partir da Etnomodelagem

Kamilo Manchego[1] & Milton Rosa[2]

## Abstract

The aim of this article is to propose ethical ethnomodels that connect the bottle cap spinning top and integral calculus. This research is theoretically grounded in the Ethnomathematics Program and Ethnomodeling. The methodology employed is qualitative in nature, based on a prior ethnographic study in which the practice of making the bottle cap spinning top was examined, and an ethnomathematical analysis was developed to propose relationships between the cultural artifact (emic perspective) and integral calculus (etic perspective). From the ethical (global) perspective of Ethnomodeling, Integral Calculus can be approached by considering the constitutive elements of the bottle cap spinning top. This may represent a proposal for future pedagogical actions grounded in cultural perspectives in Calculus classrooms.

**Keywords:** Ethnomodeling. Ethnomathematics. Integral Calculus. Mathematics Education. Bottle Cap Spinning Top.

## Resumen

El objetivo de este artículo consiste en proponer etnomodelos éticos que vinculen al Trompo de Tapitas y al cálculo integral. Esta investigación se fundamentó teóricamente en el Programa Etnomatemáticas y la Etnomodelación. La metodología empleada es de tipo cualitativo basada en un estudio previo de carácter etnográfico, en donde se hizo un estudio a la práctica de la elaboración del Trompo de Tapitas y se desarrolla un análisis Etnomatemático para proponer relaciones entre el artefacto cultural (perspectiva émica) y el cálculo integral (perspectiva ética). Desde el punto de vista ético (global) de la Etnomodelación, se podría abordar el Cálculo Integral al tener en cuenta los elementos constitutivos del Trompo de Tapitas. Lo anterior puede representar una propuesta para futuras acciones pedagógicas basadas en perspectivas culturales en aulas de Cálculo.

**Palabras clave:** Cultura. Etnomatemáticas. Etnomodelación. Cálculo Integral. Trompo de Tapitas.

## Resumo

O objetivo deste artigo consiste em propor etnomodelos éticos que vinculam o Pião de Tapinhas ao cálculo integral. Esta pesquisa fundamenta-se teoricamente no Programa Etnomatemática e na Etnomodelagem. A metodologia empregada é de natureza qualitativa, baseada em um estudo prévio de caráter etnográfico, no qual se realizou uma investigação sobre a prática de elaboração do Pião de Tapinhas e se desenvolve uma análise etnomatemática para propor relações entre o artefato cultural (perspectiva êmica) e o cálculo integral (perspectiva ética). Do ponto de vista ético (global) da Etnomodelagem, o Cálculo Integral pode ser abordado ao se considerar os elementos constitutivos do Pião de Tapinhas. O exposto pode representar uma proposta para futuras ações pedagógicas fundamentadas em perspectivas culturais em salas de aula de Cálculo.

**Palavras-chave:** Etnomodelagem. Etnomatemática. Cálculo Integral. Educação Matemática. Pião de Tapinhas.

[1] Licenciado em Matemática da Universidad del Atlántico (UA), Colômbia. Pós-graduando do Programa de Mestrado em Educação Matemática da Universidade Federal de Ouro Preto (UFOP), Ouro Preto, Minas Gerais, Brasil. E-mail: kamilo.palacio@aluno.ufop.edu.br

[2] Doutor em Educação-Liderança Educacional pela Universidade São Paulo (USP). Professor Associado IV da Universidade Federal de Ouro Preto (UFOP), Ouro Preto, Minas Gerais, Brasil. E-mail: milton.rosa@ufop.edu.br

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## 1. Introduction

Today, a considerable number of researchers in mathematics education have adopted an intercultural positioning and rely on the *Ethnomathematics Program*, because they allow them to propose relationships between practices and *cultural artifacts*[1] with the content of academic mathematics (D'Ambrosio, 2014; Rodríguez-Nieto, 2020, 2021; Rosa; Orey, 2017b; Manchego; Utria; Aroca, 2024). This generates the implicit, and sometimes explicit, purpose of showing and valuing the mathematics people use in their daily practices.

This purpose is evident in various studies in mathematics education that seek to offer alternatives to the content of academic mathematics (D'Ambrosio, 2001, 2002, 2019). These alternatives include, for example, mathematical modeling from a sociocultural perspective (D'Ambrosio, 2001, 2014; Gerdes, 2013; Rodríguez-Nieto, 2020, 2021; Rosa; Orey, 2017b; Torres; Vanegas; Aroca, 2023; Ruiz; Pacheco; Aroca, 2023; Cantillo; Mojica; Aroca, 2022).

This study is relevant from the perspectives of mathematics education, ethnomodelling, and the Ethnomathematics Program, as it provides material that can inform the design of math assignments, activities, and lesson plans. For the purposes of this study, these contributions are oriented to the design of contents associated with integral calculus, which represent an opportunity to learn about aspects of a specific cultural artifact, the trompo de tapitas (bottle cap spinning top), as well as the practices that derive from said artifact (D'Ambrosio, 2001, 2014, 2019; Rosa; Orey, 2017b, 2017a; Manchego; Utria; Aroca, 2024).

Therefore, this study presents moments of how the interviewees design this artifact, how they use measures, parts of the languages used to explain specific situations, translations constructed by researchers from the emic to the etic, among other aspects (D'Ambrosio, 2001, 2014; Rodríguez-Nieto; Escobar-Ramírez, 2022; Manchego; Utria; Aroca, 2024).

In general, this research potentially reinforces the valuation of sociocultural diversity, seeks to make visible the mathematical knowledge, know-how, and skills of the cultural group studied, and investigates the potential that a proposal integrating culture into spaces such as university education, especially in calculus courses, can offer. Thus, we present an analysis of a cultural artifact (trompo de tapitas/ bottle cap spinning top) and the mathematical practices used to create said artifact. In addition, calculus teachers are provided with elements that could be problematized in calculus courses.

## 2. What is the Bottle Cap Spinning Top?

To offer a conceptualization as approximate as possible about the bottle cap spinning top, in the first instance, a conceptualization of what is considered a spinning top will be offered. The spinning top, also called in other regions and countries a *peonza* or *pião*, is one of the oldest toys in human history (dating back more than 6000 years) and has appeared in numerous ancient cultures (Öfele, 1999; Conejo, 2018).

In relation to the characteristics of the spinning top, this is a handmade toy that commonly has a tip located at its center of gravity (Herrador, 2022). Spinning tops can have different designs, tips of different lengths and bodies of various shapes and sizes; for this reason, depending on their design, they can be used with or without a rope (Manchego; Utria; Aroca, 2024; Utria; Manchego, 2024; Utria; Manchego; Aroca, 2024). In addition, when this device is used, it can be rotated on the ground, on a rope, and/or in the palm of the hand (Öfele, 1999; Conejo, 2018).

[1] Cultural artifacts are the material manifestations of culture. They correspond to those aspects of the material technology of a group that allow it to satisfy its basic needs for food, shelter, transport and the like. Land use systems and agricultural production are cultural artifacts, as are tools and a particular design of a dress (Gavarrete, 2015, p. 152).

It is also common to find children in various regions of Colombia playing with this toy, watching it spin, competing, watching which spinning tops spin the longest and/or developing more structured games (Ardila B., 2021). Most of the time, when children play in groups, they determine the rules and organize the playing field with the purpose of playing without unforeseen events or irregularities (Manchego *et al.*, 2024). See Figure 1 for a spinning top.

**Figure 1** – Bottle cap spinning top (trompo de tapitas) made in Soledad, Atlántico, Colombia

**Source:** Personal archive of the researchers

For all the above and taking into account the observation of the cultural artifact called *trompo de tapitas*, as well as the respondents' explanations in the interview, the object is a handmade spinning top, peonza or pião. Its body is made with two plastic soft drink or soda caps, and its tip is made of a lollipop or popsicle plastic stick, positioned in the center of one of these caps. This toy is made in some regions of the Colombian Caribbean coast, and its purpose is recreation and play on the streets of the children's neighborhoods.

## 3. The Ethnomathematics Program

At present, research on the integration of sociocultural aspects in mathematics education has led to an interest in grounding it theoretically in the Ethnomathematics Program (Ávila-Toscano *et al.* 2023). This has led several authors to suggest that this research program be theoretically based, since it values and respects the mathematics present in various sociocultural practices. In addition, these studies provide a range of possibilities for connecting the mathematics of locally developed practices and school mathematics (Aroca, 2022; D'Ambrosio, 2021; Rosa; Orey, 2017b; Rodríguez-Nieto, 2020, 2021).

Within the diversity of studies associated with the Ethnomathematics Program, we can find works such as research carried out by:

a) Gerdes (2013) explores how the Bora reflect their worldview, their traditional mathematical knowledge, their relationship with nature in the baskets woven by their community, and the importance of preserving and valuing this ancestral knowledge, both culturally and educationally.
b) Rosa (2020) proposes that glocalization corresponds to the intersection between global and local knowings and doings through a dialogical relationship, and proposes a discussion between local, global and glocal mathematical approaches for the development of research in ethnomodelling.

c) Manchego, Utria e Aroca (2024) explore sociocultural practices, propose lesson plans from a parallel and comparative perspective and develop classroom activities based on ethnomathematical connections.

The studies described represent a small part of the bibliographic production that has been approached theoretically from the Ethnomathematics Program and the vast majority show relationships between the mathematical ideas present in local practices with globalizing mathematics (D'Ambrosio, 2021; Rodríguez-Nieto, 2021; Gerdes, 2013; Rosa; Orey, 2017a; Rodríguez-Nieto, 2020; Rodríguez-Nieto; Escobar-Ramírez, 2022; Rodríguez-Nieto *et al.* 2022).

In this sense, it is considered that the Ethnomathematics Program values, respects, and shows the mathematical knowledge, knowings, and doings present in sociocultural groups. This represents an innovative aspect from the perspective of mathematics education, since it allows local practices to be valued and respected, and enriches the teaching and learning processes of mathematics (Gerdes, 2013; D'Ambrosio, 2014; Aroca, 2022; Marchon, 2021).

From an educational perspective, the Ethnomathematics Program offers an alternative in which mathematics can be developed in a transdisciplinary way and can promote the development of meanings in the learning of mathematics (Aroca, 2022; Rosa; Orey, 2017a; Manchego; Utria; Aroca, 2024)

This research recognizes that ethnomathematics corresponds to a scientific research program in a Lakatosian sense that was established in the 1970s as a new perspective in mathematics education by integrating the cultural, political, social, and economic forces that shape the world (D'Ambrosio, 2021; Paixão *et al.* 2025).

As described, ethnomathematics corresponds to a research program that epistemologically and etymologically respects, values, and studies the modes, styles, arts, and techniques (*technés* or *ticas*) employed to explain, learn, know, communicate and/or deal with (*matema*) natural, social, cultural, and imaginary environments (etnos) of a sociocultural group, that is, *EtnoMatemaTica* can be interpreted broadly as the *ticas* of *matema* in a certain *etno* (D'Ambrosio, 2001, 2002, 2014, 2019).

## 4. Ethnomodelling

As described in the conceptualization of ethnomathematics, this research program offers a holistic view of mathematical knowledge, knowings, and doings, which leads to the fact that, often, in mathematics education, *ethnomodels* are used to establish relationships between mathematical knowings developed in a local context and global mathematical knowings (Cortes, 2017; Rosa; Orey, 2017b; Rosa, 2020).

*Ethnomodels*, according to Rosa e Orey (2017b), can be understood as units of information that represent mathematical systems of reality and help members of different cultural groups understand and appropriate their activities as they develop their mathematical practices. In this sense, *ethnomodels* are emic (local), etic (global) and dialogic (glocal) (Rosa; Orey, 2017a). The three types of *ethnomodels* are described:

- *The emic (local) ethnomodels*: For Rosa e Orey (2017b) and Cortes (2017), these ethnomodels are based on the mathematical information that is important for the members of a specific sociocultural group, since they represent the mathematical thinking of said members and seek to translate their way of thinking mathematically.
- *The etic (global) ethnomodels*: For Rosa e Orey (2017b) and Cortes (2017), they are mathematical constructions based on the vision of external observers, usually coming from the academy, about the reality that is being modeled.

– *The dialogic (glocal) ethnomodels*: For Rosa e Orey (2017b) and Cortes (2017), they correspond to mathematical ideas and procedures that can dialogue between different systems of knowings or knowledge and that allow a translation between said systems.

In a similar way, we proceed with the concept of ethnomodelling, which is nothing more than the action of modeling or building ethnomodels in emic (local), etic (global) and dialogic (glocal) contexts (Rosa; Orey, 2017b). In a formal way, ethnomodelling conceptually emerges from the intersection between cultural anthropology, ethnomathematics and mathematical modelling from a sociocultural perspective (Rosa *et al.* 2022).

For Rosa e Orey (2017b), Cortes (2017) and Rosa *et al.* (2022), ethnomodelling focuses on solving problems of daily life, in which the understanding of local mathematical systems is initially taken into account. This is intended to help students and teachers understand the importance of mathematics in their sociocultural context.

For this reason, ethnomodelling makes it possible to translate local mathematical ideas (emic) into a global mathematical, school or academic language (etic), or vice versa. In the Table 1, the local approach (emic), the global/academic approach (etic), and the glocal approach (dialogic) are presented, according to Rosa e Orey (2017b).

**Table 1** – Emic, etic and dialogic approach to ethnomodelling and its perspectives

| Approach | Perspective | Description |
|---|---|---|
| Emic | Local | It refers to the mathematical knowledge, knowings, and doings developed by members of a specific cultural group when performing a certain practice or elaborating a cultural artifact in which, in a consensual manner, their worldviews, cosmogonies, and cosmologies are maintained. |
| Etic | Global | It refers to a vision external to a culture, but which, from an academic perspective, could also be considered the scientific vision of mathematics that researchers interpret in relation to mathematics developed locally by other cultures. |
| Dialogic | Glocal | It refers to the translation between local (emic) mathematical knowledge, knowings, and doings and a global/academic mathematical language and/or vice-versa. This approach seeks to ensure dialogue, respect, and/or appreciation between the cultures involved. |

**Source:** Adapted from Rosa e Orey (2017b).

In this sense, both the emic approach and the etic approach play an important role in research in ethnomodelling and are important for mathematics education as a field of knowledge, since they help to understand not only aspects that are traditionally communicated from mathematics, but also more local aspects that have historically been ignored or undervalued (Rosa; Orey, 2017b).

In the same way, the *dialogic* approach constitutes a crucial element in mathematics education as a field of knowings and of doings. This approach recognizes that it is not enough to communicate only the *local*; the same happens with the "*global*", that is, it does not ignore the existence of cultural dynamics and the search for transdisciplinary and transcultural alternatives in mathematics education (Rosa; Orey, 2017b, 2017a).

Mathematics education is mobilized between diverse cultural dynamics in which, invisibly and sometimes indirectly, power relations are manifested (Rosa; Madruga; Pinheiro, 2024). The foregoing implies that mathematics educators must recognize that mathematics education is a human issue and, for this reason, they must mobilize their practice to the *glocal*, the *dialogic* in order not to deny the diverse realities of the classrooms and not fall indirectly into colonialist actions, impositions, dogmatisms, and/or ethnocentrisms (Dussel, 1977; D'Ambrosio, 2001, 2014, 2019; Campos, 1991; Aroca, 2022; Rosa; Orey, 2017b, 2017a, 2024b).

For all these reasons, this study, when approached theoretically from the perspectives of the Ethnomathematics Program (D'Ambrosio, 1985, 2001, 2014, 2019) and ethnomodelling (Rosa, 2010; Rosa; Orey, 2017b, 2017a, 2024b), allows the problems raised regarding the analysis of mathematical knowledge, knowings, and doings that have been used in the production of the spinning top, to open the possibility of proposing etic ethnomodels in the understanding of the elements of this artifact from an etic approach. Thus, the proposed ethnomodels can help students perceive the relevance of mathematics and, in parallel, help teachers value, preserve, and respect other cultures.

For this reason, the objective of this work has been to propose etic ethnomodels that link the spinning top and the integral calculation. The dialogic perspective is evident but leans towards an etic understanding of the emic in this article. The authors are aware that the dialogic relationship could also manifest in the opposite sense (emic understanding of the etic) or as a balanced, horizontal dialogic relationship, but these two other relationships will not be presented in this article. If you are interested in a more balanced dialogic analysis, you can consult Manchego, Utria e Aroca (2024) or, if you are associated with a dialogic analysis more from the emic to the etic, you can consult Manchego Palacio e Rosa (2026).

## 5. Methodology

Methodologically, this article will be presented in two moments: a) the first corresponds to the description of how a group of children elaborates the bottle cap spinning top and an ethnomathematical analysis will be developed and b) the second corresponds to the presentation of the relationships between the knowings and doings immersed in the artifact that emerged from local practice (*trompo de tapitas*) and the "global" mathematical knowing (approached from the integral calculation).

Therefore, the methodology used is qualitative, based on an ethnographic study (Rueda, 1999; Hernández; Fernández; Baptista, 2014; Gildersleeve; Kuntz, 2011; Angrosino, 2007). It is ethnographic because the strategies and instruments that ethnography offers were used to produce and analyze data, and because it was carried out over a short period of time (Rueda, 1999; Silva; Burgos, 2011). This study was conducted using this approach because the researchers examined the preparation of the bottle cap spinning top by children, using instruments for qualitative ethnographic research (Manchego; Utria; Aroca, 2024).

According to Loayza-Maturrano (2020), "qualitative research focuses on the understanding of phenomena and can focus on meanings, perceptions, concepts, thoughts, experiences, or feelings. Qualitative research examines how or why a phenomenon occurs" (p. 57). This order of ideas is followed in this study, as we conducted semi-structured interviews and used a field diary, along with photographic and audiovisual records, to understand participants' meanings, perceptions, concepts, thoughts, and experiences (Aroca, 2022; Manchego; Utria; Aroca, 2024)[2].

For the above, methodological instruments provided by ethnography were used to analyze the practice developed by children when making the spinning top and with the purpose of proposing ethnomodels built from the knowledge, knowings, and emic doings and the knowledge, knowings, and etic mathematical doings that surround the cultural artifact.

[2] In the study carried out by Manchego, Utria e Aroca (2024), a methodology based on three stages is presented. The first corresponds to an ethnographic study of the construction and play of the spinning top; the second corresponds to the design of a class plan to incorporate the spinning top in the classroom under a framework of dialogue and respect and the third corresponds to the exploration of ethnomathematical connections built by students when implementing various activities in mathematics classes in a public school in Barranquilla.

## 5.1 Instruments and strategies for data collection

Initially, *participant observation* was used because it involves the sociocultural interaction between the researcher and the people who are part of the environment being investigated, where the researcher collects the information without being intrusive and writes the records in the *field diary* according to the objective of the research (Taylor; Bogdan, 1984; Manchego; Utria; Aroca, 2024).

This helped researchers know and understand the knowledge, knowings, and doings that surround the spinning top and to understand the cultural artifact in its constituent elements to subsequently analyze said knowledge, knowings, and doings and the cultural artifact. Given the above, the mathematical knowledge, knowings, and doings of the interviewees were analyzed when explaining how the spinning top is made and what the purpose of each of its components is.

Importantly, this cultural artifact is manufactured by children aged 8 to 12 years old. The children interviewed are located in a neighborhood of Soledad, Atlántico, Colombia and for data collection, production, and analysis, the respective permissions were requested from their guardians, who declared that such data can be used exclusively for academic and non-commercial aspects and verbally reported that if the products contribute socially and academically, they can be published without inconvenience (Manchego *et al.*, 2024).

Consequently, we could resume the *semi-structured interview*. This interview was conducted to understand what the interviewees thought when they developed their practices, the verbal language used in the practice, and their explanations of the construction process. According to Kawulich (2006), the *semi-structured interview* is used to know what people think from pre-established questions, while the researcher engages with people, processes, and their culture.

Therefore, this study identifies aspects of the practice that revolve around the assembly of the spinning top and its constituent elements and explores how respondents describe their practice. The *participant observation* applied in parallel with the development of the *semi-structured interview* and an *audiovisual production* made it possible to pose pre-established questions (flexible according to the context), adapt the questions *in situ* and record through photos and videos the processes of constructing the spinning top and the analysis of its elements (Aroca, 2022; Manchego; Utria; Aroca, 2024).

The *audiovisual record* (photos and smartphone videos) was applied to parts of the interviews and the production processes. According to Aroca (2022) e Manchego, Utria e Aroca (2024), the audiovisual data collected facilitated the researchers' analysis of sociocultural practice and enabled them to compare aspects of the practice with mathematical content in schools and universities.

## 5.2 Data analysis

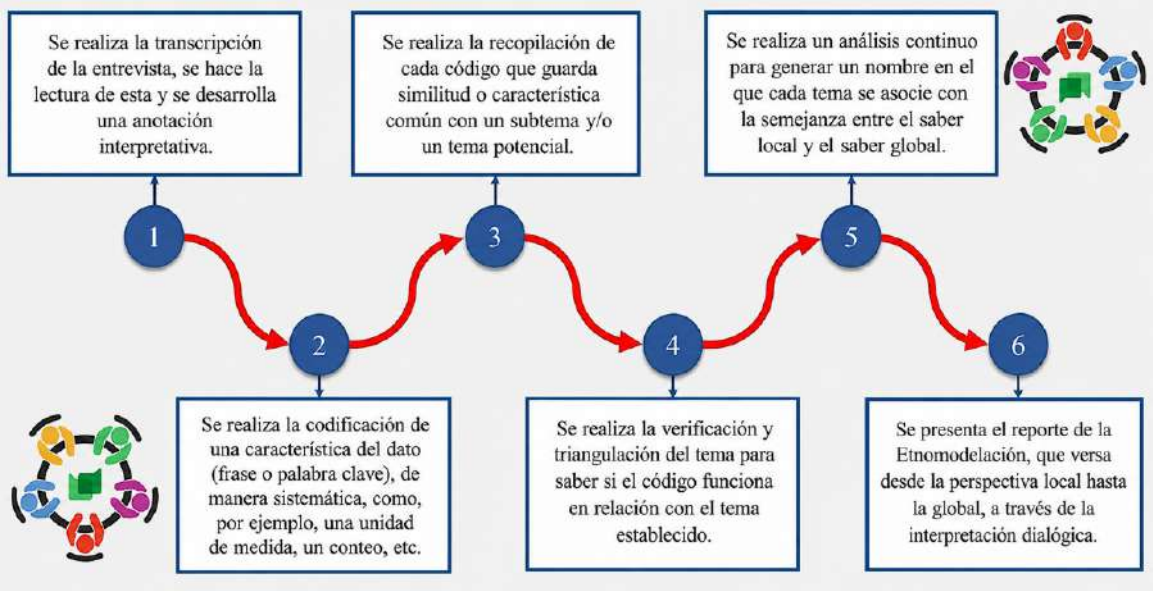
For the analytical process, we used thematic analysis and analysis in ethnomathematics (D'Ambrosio, 2014), since these methods of analysis allow identifying, organizing, describing, analyzing, and minimally reporting patterns within the data collected (Braun; Clarke, 2006). The analysis in ethnomathematics is taken into account, because this type of analysis seeks to describe and understand the knowledge, knowings, and doings developed by sociocultural groups based on observations and, in addition, because the research developed from the Programa Etnomatemática offers the opportunity to observe the practices of different cultural groups, followed by an analysis of what they do and why the group members do it (D'Ambrosio, 2014, 2019).

This analytical process was carried out in six phases: 1) in the *first*, we made the transcription of the interview, read it and made took interpretive notes; 2) the *second* corresponds to familiarization with the data and the generation of initial codes; 3) the *third* corresponds to the collection of codes that have similarities or common characteristics with subtopics and/or potential topics; 4) the *fourth* corresponds to the verification and triangulation of the topics to know if the codes work in relation to the established topics; 5) the *fifth* corresponds to continuous analysis, generating names in which each

topic is associated by the similarities between local and global knowledge and; 6) the *sixth* corresponds to the production of a report.

In general, the analysis was organized in two parts: a) the *first* began with the exploration of sociocultural practice and the artifact that emerges from the practice (study with ethnographic characteristics), and b) in the *second*, a thematic analysis of cultural practice based on theoretical bases of ethnomodelling and its relationship with globalizing mathematics was developed. Figure 2 shows the phases of the thematic ethnomathematical analysis.

**Figure 2** – Phases of the thematic ethnomathematical analysis




**Source:** Own elaboration based on Braun e Clarke (2006)

This analytical process, from the perspective of Braun e Clarke (2006), can be interpreted as a type of deductive analysis whose development is theoretically grounded in ethnomodelling and the Ethnomathematics Program. Both theoretical bases are of interest to researchers and, at present, are promoted by other researchers and analysts in different parts of the world.

## 6. Results and discussions

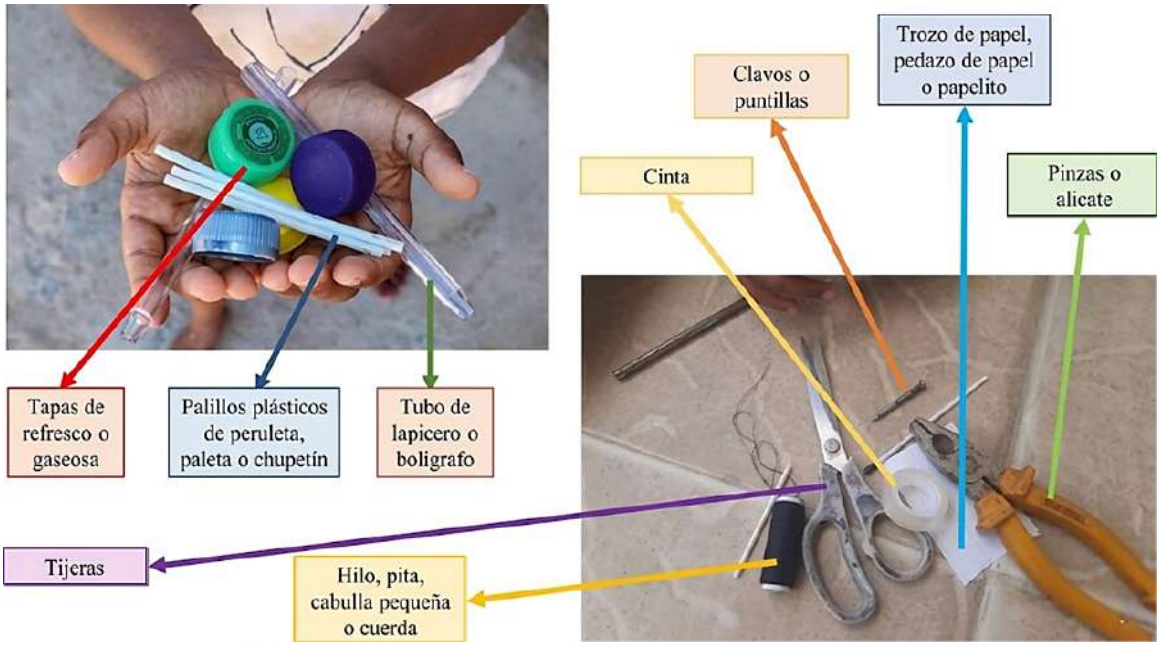
In this section, the results and discussion are presented in three moments that emerged from the ethnomathematical analysis of the children's construction of the bottle cap spinning top and from the thematic analysis.

1) *Moment 1: result of the ethnomathematical analysis of the practice.* We describe the process of manufacturing the bottle cap spinning top and propose an ethnomathematical analysis of the practice.
2) *Moment 2: proposal of etic ethnomodels.* We propose developing etic (global) ethnomodels based on the components that make up the spinning top. The (global) etic ethnomodels are developed from the "global" academic perspective of integral calculus and include a set of exercises that could be used as activities in a calculus course.
3) *Moment 3: Discussion.* We proposed a discussion of the results and their possible contribution to the integral calculus.

## 6.1 Moment 1: results of the ethnomathematical analysis of the practice

When analyzing the practice of making the bottle cap spinning top, we found that children look for reusable materials such as juice or soda caps, straws, lollipop stickers, and pencil tubes (Manchego *et al.*, 2024). Likewise, to build the spinning top, children use tools such as scissors, tweezers, thread, a nail, tape, a piece of paper, and, on some occasions, a hammer (Manchego; Utria; Aroca, 2024). Figure 3 shows the materials and tools used.

**Figure 3** – Materials and tools used




**Source:** Personal archive of the researchers

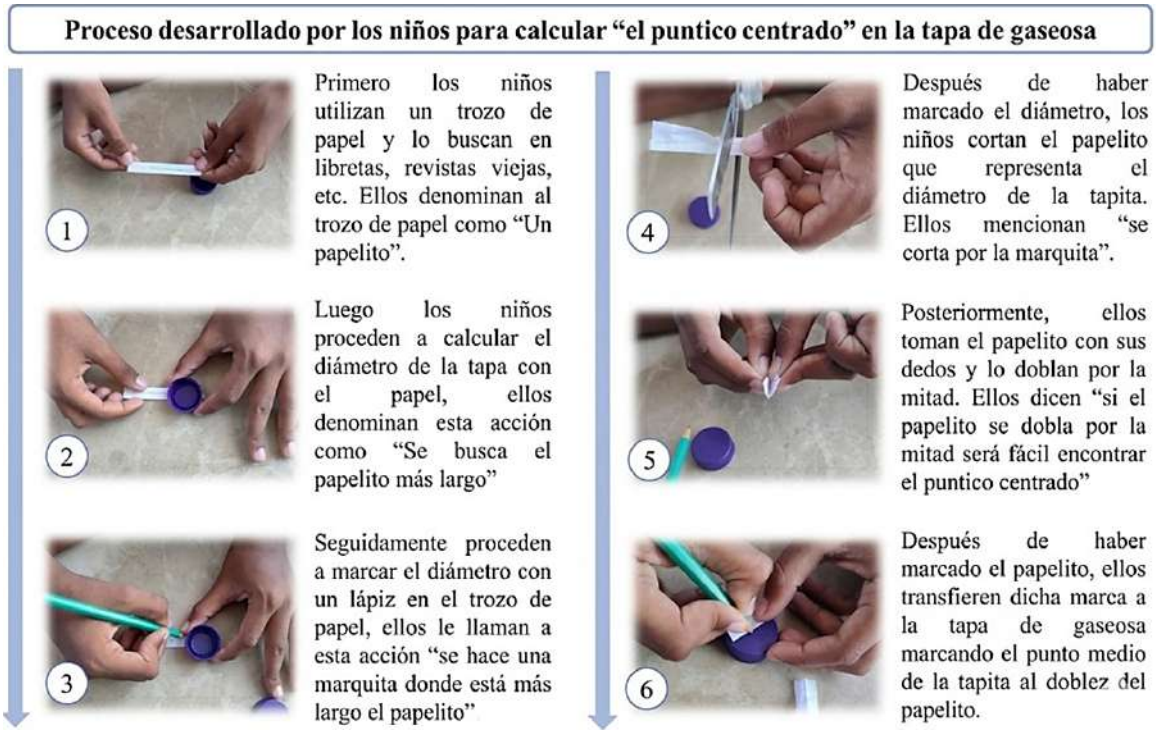
The data from the interview and the audiovisual production helped identify that the infants use techniques that allow them to calculate the center of the soda caps when using a piece of paper, or, as they say, *papelito*. We noticed that they transfer the diameter of the lid onto the piece of paper by comparison (Manchego; Utria; Aroca, 2024).

Subsequently, to find the half or midpoint of the soft drink lid, the kids fold the paper they used in half. In the interview available in Matemáticas del Pueblo — People's Math (2022), Johan says that: "If you see there, the piece of paper is the same size as a checkered board part [...]. And there, we have to mark the half that would come out here". Figure 4 shows the children measuring the diameter of the soft drink lid and marking its center.

The interpretation of this information from academic mathematics (global/etic) enables the translation of the mathematical knowledge, knowings, and doings present in practice into categories typical of school and university mathematics. However, this *translation* does not imply the substitution of (local) emic knowledge, knowings, and doings for academic knowledge, but the establishment of a dialogue between different knowledge systems. In other words, this process is not free of epistemological tensions, since any *translation* between different knowledge systems risks simplifying or reinterpreting sociocultural meanings relevant to the members of the cultural group studied (Rosa; Orey, 2017b, 2024a).

In this sense, an etic (global) interpretation suggests that the children identify a relationship between the radius of the soft drink lid and the act of folding the *papelito* in half. This emic (local) measurement procedure can be related to formal geometric concepts, without losing sight of the sociocultural meanings it acquires within the practice studied. At this time, it is possible to etically interpret

**Figure 4** – Process developed by the children to calculate "the centered point"on the soft drink lid




**Source:** Personal archive of the researchers

that there is an awareness of the diameter of the lid when the paper is unfolded and cut based on a comparison process with the soft drink lid when the paper is longer.

In the same way, etically, it is also possible to interpret that the center of the soft drink lid (the edge of the lid could be interpreted as a circumference and the base of the lid as a circle) emerges after having folded the paper and marked the lid (thanks to the fold made in advance) with a pencil. After transferring the mark from the folded paper to the soft drink lid, the children proceed to pierce the lid at the mark made.

The described resolution technique shows that the perforation is made in the center of the soft drink lid with a nail, and consequently, the lollipop stick is inserted through the hole. The straw will be the tip of the toy. The children report that the tip of the spinning top must be two fingers long.

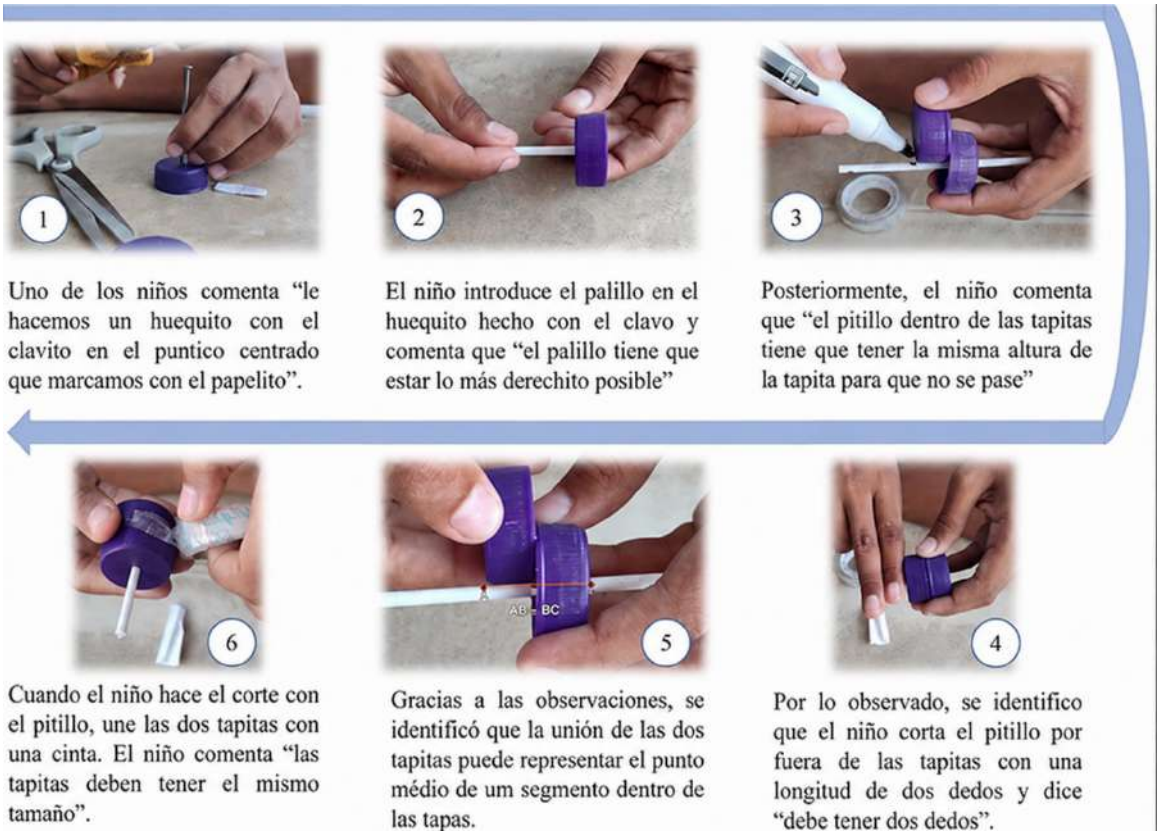
For this reason, the translation developed in ethnomodellation should not be understood as a substitution of (local) emic knowledge, knowings, and doings for academic knowledge, but as a dialogical process that seeks to establish relationships between different ways of producing knowledge.

In this sense, an etic interpretation suggests that the children identify relationships between the diameter, the radius, and the center of the soft drink lid when they fold the *papelito* in half, compare its length with the lid, and transfer the mark obtained to make the perforation. These local measurement and location procedures can be brought into dialogue with formal geometric concepts without losing sight of the sociocultural meanings they acquire within the practice under study.

Figure 5 shows the manufacturing of the bottle cap spinning top using a locally developed mathematization process.

The interpretation of these results shows that the children use mathematizing techniques developed locally (emically) and are naturally found in their ethnomathematics. In this context, Rosa e Orey (2017b) highlight that the mathematization of reality is elaborated by members of different cultu-

**Figure 5** – Construction of the bottle cap spinning top through a local mathematization process




**Source:** Personal archive of the researchers

ral groups through representations of the sociocultural context itself, which is generated via inferences, communication, and the use of mental models based on local mathematical *knowings* and *doings*.

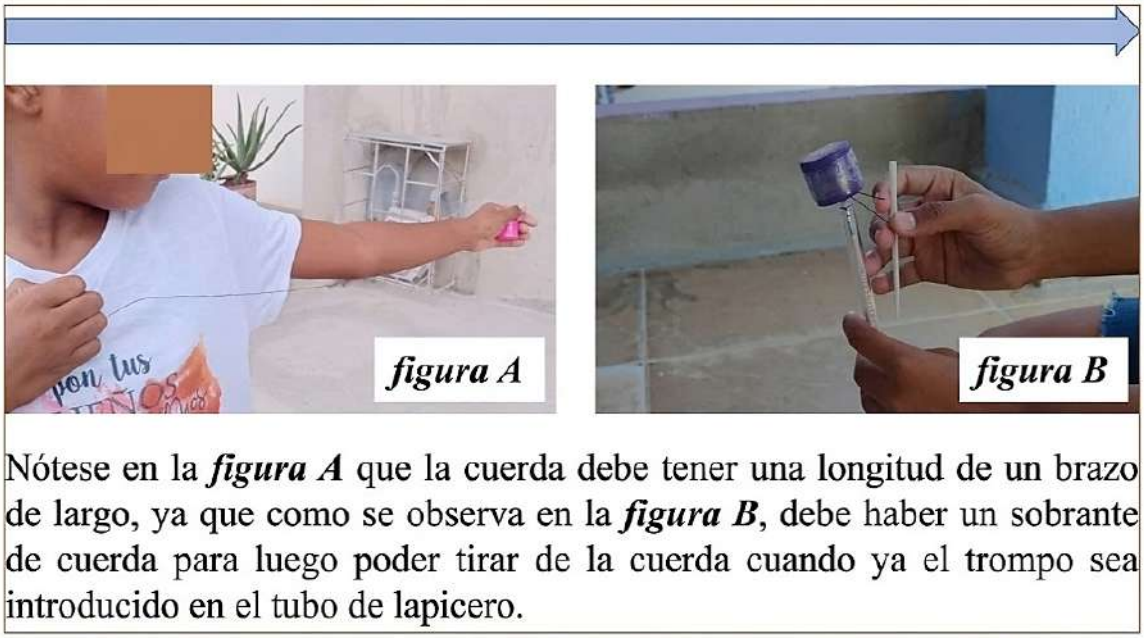
After joining the two soft drink lids with tape, the children measure the rope that will be used to spin the top. They mention that the rope must be "one arm long [...] because if it is shorter, it is not enough, and if it is longer, it gets entangled."The interpretation of this local (emic) mathematical *knowing/doing* shows that, according to the children, the appropriate length of the rope for this game is one arm long, since being longer or shorter will make it difficult to spin.

After calculating the length of the rope, the children proceed to wind the rope from the bottom up at the tip of the spinner and comment that "when it is rolled up, the tip of the spinner is inserted into the small hole of a pencil shell"; that is, the children insert the tip of the spinning top into the hole of a pencil tube. It is important to note that the rope must be wound from the bottom up; if it is wound in the opposite direction, the top of the pencil tube will not come out. Figure 6 shows the length of the string and the spinning top ready to be thrown.

The interpretation of these data shows that the local (emic) mathematical knowledge of the children in the construction of the spinning top are related, according to Rosa and Orey (2012), to the accounts, descriptions, and analysis of the mathematical practices that are expressed in terms of the categories and conceptual schemes that they consider significant and appropriate for use in the activities present in their own sociocultural context.

According to this emic (local) approach, Rosa e Orey (2017b) argue that this mathematical knowledge aligns with the perceptions and interpretations deemed appropriate by members of distinct cultural groups within their own cultures, through an internal vision of how this knowledge is constructed.

Figure 6 – Length of the string and the spinning top ready to be thrown




Source: Personal archive of the researchers

As mentioned before, this article will place greater emphasis on the components of the spinning top. This means that the elements that compose it (the researchers' focus on this article) align with the article's objective. In this sense, in the following section, an etic ethnomodeling (translation) will be presented (from the integral calculation) between the constituent elements of the cultural artifact and respondents' expressions.

## 6.2 Moment 2: Proposal for the development of etic ethnomodels

Before starting with the *translation* of local (emic) mathematical knowledge, knowings, and doings into the mathematical language of other knowledge systems, such as, in this case, globalizing mathematics (etic), it is important to remember that this *translation* process poses epistemological challenges, since the interpretation of knowledge produced in specific sociocultural contexts through academic categories can generate displacements, reconfigurations or partial losses of the meanings attributed to them by their own carriers (**campos1991** ; Rosa; Madruga; Pinheiro, 2024).

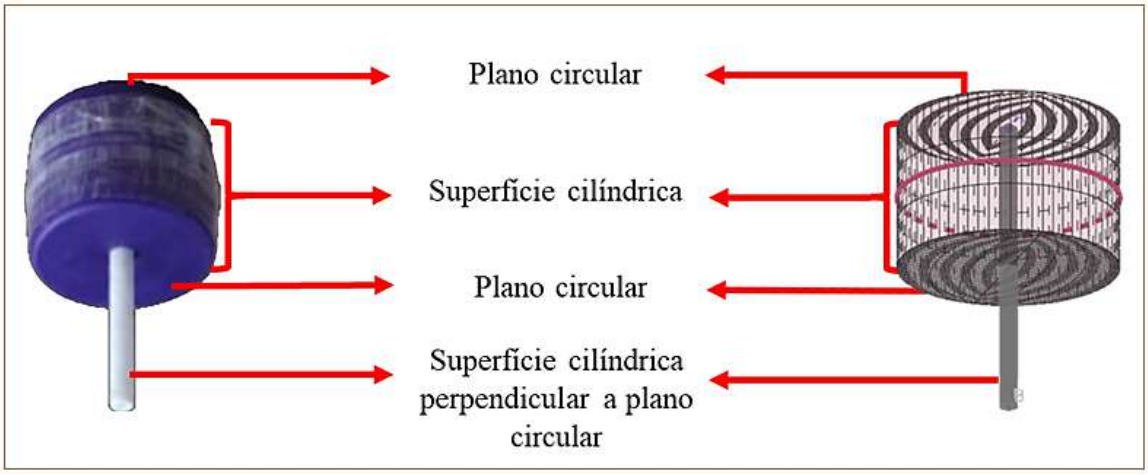
Under these considerations, and now, proceeding with the *translation*, we could interpret this as the players calculating the stick's perpendicularity to the *circular plane* of the upper part of the soft drink lid. This interpretation owns to the expression "the straw must enter as straight as possible through the lid" that emerged during the process of making the spinning top, specifically, when the plastic stick (which is part of the constituent elements of this cultural artifact) was introduced into the hole made in one of the soda caps.

The interpretation of this information also shows that, due to their physical characteristics, plastic sticks have a close relationship with cylinders, and the soda caps (part of the constituent elements of the artifact) also have a close relationship with cylindrical surfaces and circular planes.

Figure 7 shows the spinning top as a cultural artifact and its connections with globalizing mathematical objects (representation of a translation between different mathematical knowledge systems).

Therefore, we infer that the spinning top game, due to its physical characteristics, could be mathematized through a cultural *translation* with school and university mathematics using integrals to

**Figure 7** – The spinning top and its relationships with globalizing mathematical objects




**Source:** By the authors

calculate the *volume* of a *cylinder* in *an interval* for the *z-axis* (height of the cylinder) very close to the interval determined by the diameter of the circular planes. The ethnomathematical analysis allowed us to identify that the *cylinder* formed by the curved surface of the soda caps has a length equivalent to *two fingers*, as does the part of the straw that extends beyond the caps.

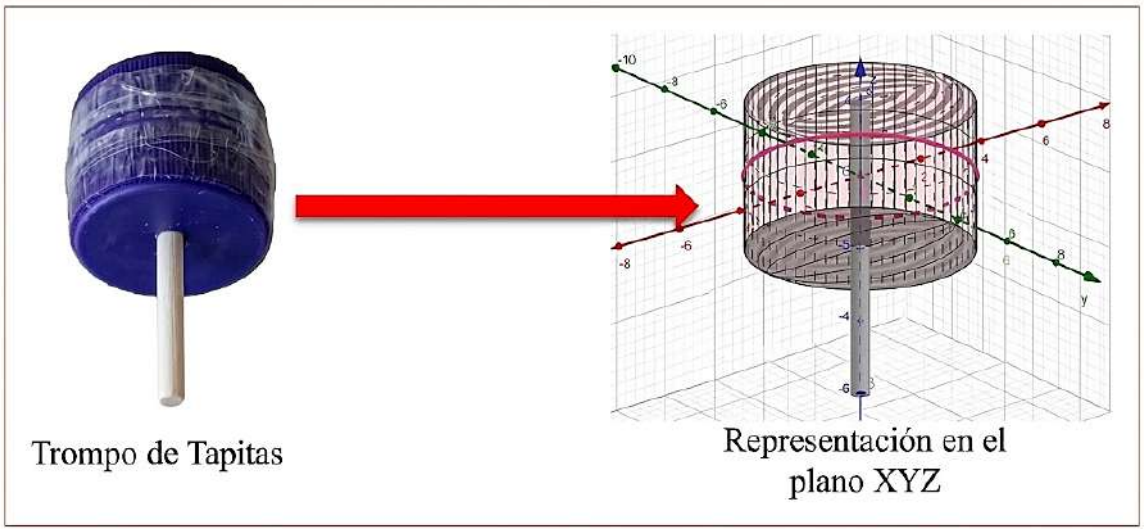
In this direction, etic (global) ethnomodels can be built to represent the *cylinder* modelling the body of the spinning top, preserving the proportional relationships derived from the *units of measurement* the children used during its construction. The conservation of these proportions allows defining the geometric dimensions of the model (particularly *the radius* and *height* of the *cylinder*) and establishing a mathematical representation consistent with the physical characteristics observed in the cultural artifact. In this way, cultural *translation* makes it possible to formulate ethnomodels based on concepts and procedures of integral calculation.

According to the proportional relationships identified in cultural practice, we assume that the unit of emic (local) measurement called *dedo* (finger) corresponds to *two units* in length in a three-dimensional Cartesian coordinate system; that is, 1 *dedo* = $2u$. In this correspondence, the height of the cylinder modelling the body of the spinning top can be represented by $h = 4u$, since the participants indicate a length equivalent to two fingers.

Likewise, given that the diameter of the soda caps is approximately equal to the height of the cylinder, we propose a geometric model whose axis of symmetry coincides with the *z-axis*, whose centre is at the origin of the coordinate system, and whose radius is $r = 2u$. These conditions make it possible to formally define a *right circular cylinder* that mathematically represents the body of the spinning top and to determine geometric properties, such as surface area and volume, using integration. Figure 8 shows the spinning top as an emic cultural artifact (local) translated under a three-dimensional geometric representation in the plane $(X, Y, Z)$.

The interpretation of the results from the ethnomathematical analysis enabled the identification of proportional relationships that serve as the basis for constructing etic (global) ethnomodels. In particular, it was observed that the height of the cylinder representing the body of the spinning top (made up of the soft drink lids and the segment of the plastic straw that passes through them) is equivalent to *two fingers* in length according to the unit of measurement used by the children. Likewise, we identified that the diameter of the cross-section of the straw corresponds approximately to the *eighth part of the diameter* of the caps used in the elaboration of the artifact. These relationships constitute local references that guide the definition of the geometric dimensions of the mathematical model.

**Figure 8** – The spinning top as a cultural artifact represented in the XYZ plane




**Source:** By the authors

Based on these interpretations, an etic (global) ethnomodel based on integral calculation is proposed to mathematically represent the volume of the spinning top body and the visible segment of the plastic straw.

According to Rosa e Orey (2017b), the elaboration of dialogical (glocal) ethnomodels requires linking the emic (local) and etic (global) perspectives, so that academic mathematical representations do not replace the knowledge developed in cultural practice but instead establish a dialogue with it.

In this sense, the locally identified measurement relationships are used as a starting point to define the geometric variables of the model and formulate mathematical procedures to determine the volume of the main components of the artifact.

Below is the formal construction of the proposed etic (global) ethnomodel.

Considering the proportional relationships identified in the ethnomathematical analysis and assuming that the center of the cylinder is located at the origin of a three-dimensional Cartesian coordinate system $(X, Y, Z)$, some correspondence is established between the local unit of measurement *dedo* (finger) and a length $n$ in said system, that is, 1 *dedo* = $n$. In this correspondence, the radius of the cylinder modelling the body of the spinning top is defined as $r_{body} = n$. Likewise, since the previous analysis showed that the diameter of the straw is approximately equal to the eighth part of the diameter of the soda caps, the radius of the cylinder representing the straw can be expressed as $r_{straw} = \frac{n}{8}$. These relationships allow establishing the fundamental geometric dimensions of the ethnomodel and constitute the basis for the formulation of analytical expressions aimed at calculating areas and volumes through integration techniques.

Under the previously established conditions, the circular base of the cylinder representing the body of the bottle cap spinning top can be described by the equation:

$$x^2 + y^2 = n^2$$

While the circular base of the cylinder representing the plastic straw can be expressed as:

$$x^2 + y^2 = \left(\frac{n}{8}\right)^2$$

Likewise, considering that the height of the body of the spinning top is equivalent to two fingers, the corresponding cylinder can be defined in three-dimensional space by the region delimited by:

$$-n \leq z \leq n$$

In this way, the height of the cylinder $h_1$ in $z$ would be $h_1 = 2n$. On the other hand, the segment of the straw that protrudes from the caps can be represented by a second cylinder of radius equal to $\frac{n}{8}$, whose height is comprised in the range:

$$-3n \leq z \leq -n$$

Thus, its length is $h_2 = 2n$. These conditions make it possible to formally define the geometric regions that constitute the body of the spinning top and the visible portion of the plastic straw, thereby forming the basis for formulating etic (global) ethnomodels to calculate their areas and volumes using integration techniques.

For what has been described, and to formulate an etic (global) ethnomodel that allows determining the volume of the main components of the spinning top through integration techniques, it is necessary to mathematically describe the geometric solids associated with this cultural artifact.

In this sense, it is not enough to establish the equations of the circular surfaces that delimit the bases of the cylinders, but it is also necessary to define the three-dimensional regions occupied by said bodies in space. This representation allows us to formally characterize the integration domains that will be used in the construction of the triple integrals associated with the calculation of their volumes. For this reason, the body of the spinning top and the visible portion of the plastic straw can be represented by the following regions of three-dimensional space:

$$C_{body} = \left\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \leq n^2, -n \leq z \leq n\right\}$$

$$C_{straw} = \left\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \leq \left(\frac{n}{8}\right)^2, -3n \leq z \leq -n\right\}$$

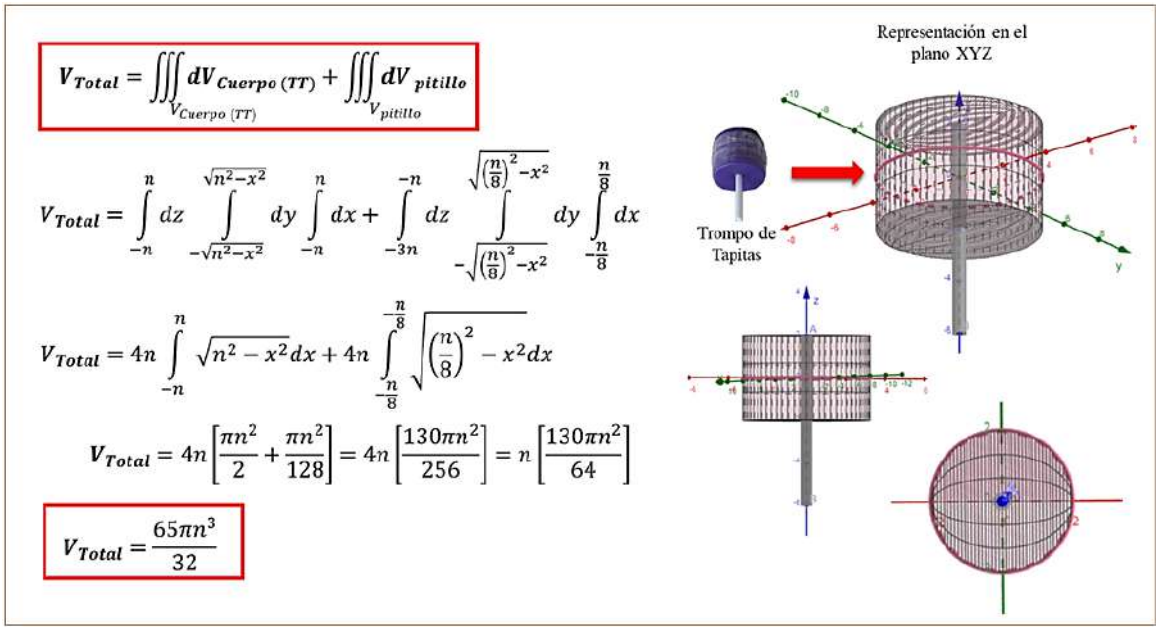
These regions constitute the integration solids that geometrically define the body of the spinning top and the exposed segment of the plastic straw, respectively, and will serve as the basis for constructing etic (global) ethnomodels to calculate their volumes using triple integrals.

Consequently, the geometric and proportional relationships identified in the ethnomathematical analysis allow us to formulate an etic (global) ethnomodeling problem for an integral calculus course, in which students should determine the volume of a spinning top using triple integrals. In a complementary way, the construction of this etic (global) ethnomodel allows us to derive a general algebraic expression for the calculation of the volume of the spinning top. This expression enables the formal representation of the artifact without delinking it from mathematical knowledge, knowings, and doings mobilized locally by the participants.

Figure 9 shows the etic (global) ethnomodel of the spinning top built from the development of a triple integral.

The proposed ethnomodel represents a general formulation built from an etic (global) approach, in which tools and concepts typical of academic mathematics are put into dialogue with the mathema-

**Figure 9** – Etic ethnomodel of the spinning top volume from the development of a triple integral

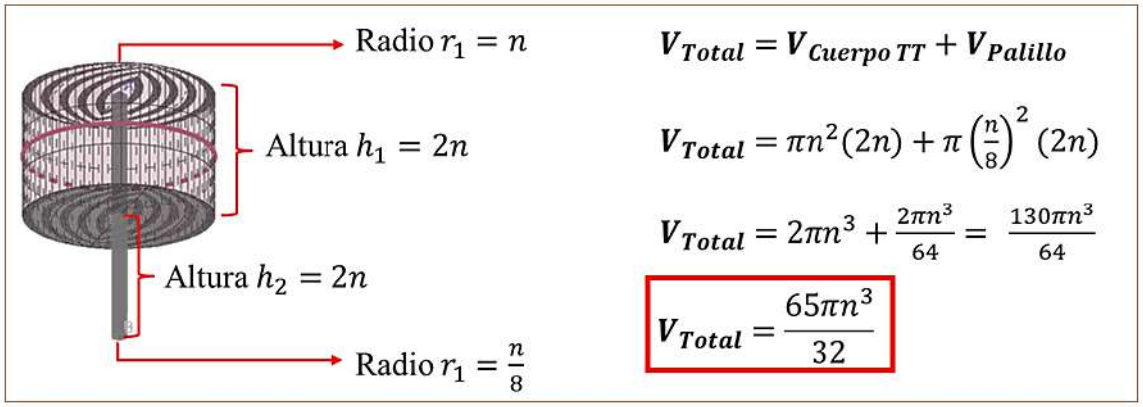



**Source:** By the authors

tical knowledge, knowings, and doings identified in the practices of the children who make the toy (spinning top). In this sense, the model does not seek to replace local ways of understanding and constructing this cultural artifact, but to offer a mathematical interpretation that allows analyzing some of its geometric characteristics through formal procedures.

Likewise, this ethnomodel can be expressed algebraically by substituting the local units of measurement identified during the ethnomathematical analysis into the formula for the volume of a cylinder. In this way, it is possible to obtain a general expression for the volume of the spinning top that preserves the proportional relationships observed between mathematical knowledge, knowings, and doings typical of the emic (local) culture and academic (etic) mathematical knowledge, knowings, and doings. Figure 10 shows the etic (global) ethnomodel of the volume of the spinning top from the algebraic development of the formula for the volume of a cylinder.

**Figure 10** – Etic (global) ethnomodel of the volume of the spinning top from the algebraic development of the formula of the volume of a cylinder




**Source:** By the authors

Consequently, the algebraic expression obtained:

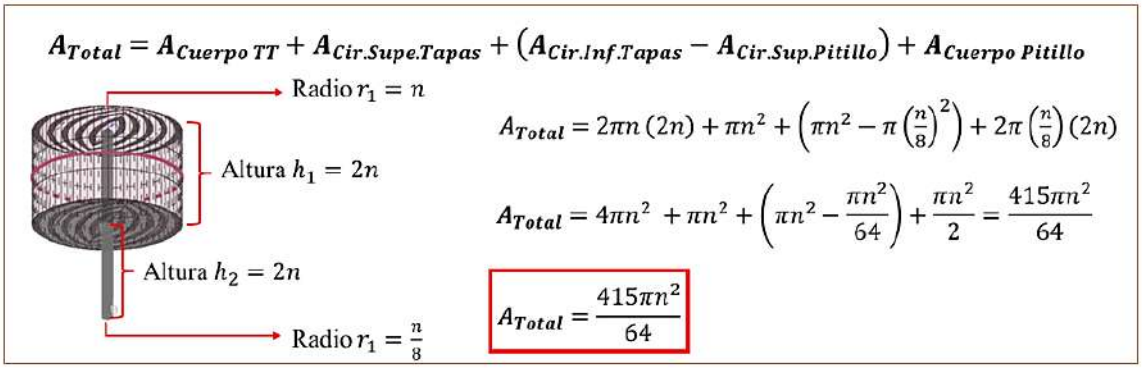
$$V_{Total} = \frac{63\pi n^3}{63}$$

[...] is a result of the process of etic (global) ethnomodelling, since it synthesizes in a single mathematical model the geometric relationships derived from the cultural practice studied.

In this order of ideas, and with the same data, the previously established geometric relationships also allow us to build etic (global) ethnomodels oriented toward studying the surface area of the spinning top. In this case, the mathematical representation of the artifact using cylindrical solids makes it possible to formulate analytical expressions to determine the areas of its different components, maintaining the proportions identified in cultural practice.

Figure 11 shows the etic (global) ethnomodel that represents the surface area of the spinning top from the algebraic development of the formula for the area of a cylinder and the area of a circle.

**Figure 11** – Etic (global) ethnomodel that represents the surface of the spinning top from the algebraic development of the formula of the area of a cylinder and the area of a circle




**Source:** By the authors

In the same way as the previous ethnomodel and consequently, the algebraic expression obtained:

$$A_{Total} = \frac{415\pi n^2}{64}$$

[...] is a result of the process of etic (global) ethnomodelling, since it synthesizes in a single mathematical model the geometric relationships derived from the cultural practice studied.

For Rosa e Orey (2017b), etic (global) ethnomodels relate to accounts, descriptions, and analyses of mathematical ideas, procedures, techniques, and practices expressed in terms of categories considered significant and appropriate by the community of external observers. This type of knowledge must be accurate, logical, complete, replicable, and independent of external observers, who have an outer, external view of the members of the cultural group studied.

The process of etic (global) ethnomodelling of this toy offers alternatives that could be problematized both in school and university contexts, since these ethnomodels are raised and problematized as a dialogical option with greater etic influence between the mathematics used in the cultural practice that revolves around the elaboration of the spinning top (emic approach) and the school mathematics associated with the calculation of volumes of geometric bodies such as cylinders (etic approach).

Thus, if these ethnomodels are raised and problematized as a dialogical option among the mathematics used in the elaboration of the cultural artifact studied with integral calculus, it would

also offer integral calculus professors alternatives of pedagogical actions in which the existence of this type of practices is communicated, at least part of its history is presented and, in addition, alternatives are mobilized for students to develop exercises with cultural and, also, mathematical-academic significance.

## 6.3 Moment 3: discussion

The results of the analysis and its interpretation show that this research can represent a contribution to ethnomodelling from an etic (global) approach to integral calculation if external observers are part of the academy and have knowledge about this discipline (Rosa; Orey, 2017b).

It is important to note that the etic (global) ethnomodels proposed in this study were neither problematized nor implemented in real teaching contexts, since the objective of this article was not to develop a pedagogical intervention in the classroom or to evaluate the didactic impact of these proposals. On the contrary, the intention was to demonstrate that it is possible to develop etic ethnomodels from local socio-cultural practices and situate them within formal academic spaces, particularly in university courses such as Integral Calculus.

From this perspective, we considered that proposals of this nature could contribute not only to the development of mathematical content traditionally addressed in such courses, but also to the promotion of intellectual curiosity, interest in understanding other ways of producing knowledge, and the valuation of sociocultural practices often absent from academic settings.

In this sense, the purpose of this work is not to instrumentalize the spinning top as a mere resource for the teaching of mathematical concepts, but to claim its existence as a legitimate cultural practice and to make visible the knowledge, knowings, and doings that emerge in its construction and use, especially in contexts where these sociocultural manifestations have been historically marginalized or made invisible by academia.

As a consequence, if the proposal regarding the development of etic (global) ethnomodels and activities derived from this process is taken into account, not only would there be an integral calculation that links real-world questions, but also an integral calculation that rethinks the possibilities of integrating cultural artifacts in their teaching and learning processes. The integration of cultural artifacts such as the bottle cap spinning top in this type of space opens the possibility of not only sharing the artifact as something to be modeled, but also the history behind it, the culture, worldviews, cosmologies, cosmogonies, ethnomathematics and ethnomodels that may emerge from its elaboration or use.

The proposal, from an academic perspective, may imply a colonial reinforcement of academic mathematics, but it is important to keep in mind two fundamental points before assuming that position: first, that ethnomathematics and ethnomodelling are not against academic mathematics. In fact, academic and school mathematics could be understood as a type of ethnomathematics in which ethnomodels typical of that mathematics are manifested (D'Ambrosio, 2014, 2019; Rosa; Orey, 2017b).

The second is that from a decolonial perspective, we do not seek to obliterate or mitigate academic mathematics (which is traditionally communicated in schools and universities); in fact, we seek mainly that local sociocultural aspects that, by hegemonic and colonialist tradition, are ignored, disrespected, and undervalued are, on the contrary, valued, respected, and be dialogically present (Campos, 1991; Dussel, 1977; Rosa; Orey, 2024b, 2024a).

In this direction, the emic (local) offers an appreciation of the mathematical techniques used in the construction of the cultural artifact (bottle cap spinning top / Trompo de Tapitas) through the use of local mathematizations from the perspective of the internal cultural dynamics and the relationships of the children with the social context in which this cultural practice is developed and, the present

work, from the etic ethnomodels, seeks to ensure that these local knowledge, knowings, and doings are not ignored, but are taken into account for the development of courses such as Calculus. A glocalized mathematical education that respects sociocultural diversity (Rosa; Orey, 2017b, 2017a, 2024a; Manchego; Utria; Aroca, 2024).

The etic (global) approach has provided a cross-cultural contrast, which employed the use of school and academic mathematical concepts in the elaboration of etic ethnomodels by using mathematical tools related to contents of integral calculus and understanding the constituent elements of the cultural artifact (*trompo de tapitas* / bottle cap spinning top) together with its ethnomathematics and ethnomodels.

It is important that an emic (local) description identifies the etic (global) characters that are locally significant, because the more we know about the etic knowledge of a certain cultural group, the easier its emic analysis will be (Rosa; Orey, 2017a, 2024a).

In other words, it is necessary that the emic description, which is locally significant, identify the etic characteristics of schools and academia, since etic (global) knowledge can be considered an external interpretation of the emic knowings and doings of the members of a certain cultural group, and not as an interpretation of culture.

To conclude this section of the discussion, it is important to consider that the dialogical (glocal) approach is associated with the acceleration and intensification of interaction and integration among members of different cultural groups. The interrelationship between practices developed in local cultures (such as those that revolve around the spinning top) and school mathematical ideas, techniques, and procedures (integral calculus) can be mutually reinforced by the use of culturally based mathematical activities that help students find the relevance of mathematics in the various cultures with respect, dialogue, and *alterity*[3].

## 7. Conclusions

For the results obtained in this study and for the fact that we were theoretically based on the Ethnomathematics Program and ethnomodelling (Manchego; Utria; Aroca, 2024; Rodríguez-Nieto; Escobar-Ramírez, 2022; D'Ambrosio, 2014, 2019, 2021; Rosa, 2010; Rosa; Orey, 2017b; Rosa; Orey; Gavarrete, 2017; Rosa *et al.* 2022), we can conclude that the constituent elements of the bottle cap spinning top can be problematized in classes of integral calculation by proposing ethnomodels that integrate the emic (local) approach of the practice and, the cultural artifact, from the multivariate integral calculation; its geometric representation and interpretation (etic and global approach) are useful to know both the artifact in question (not only from the physical, but also from the historical, from the cultural and social) and the fact of delving into issues such as the calculation of volumes from the triple integrals and their understanding from the geometric representation in space (dialogical perspective). As for the algebraic representations that generalize the area and volume for the cultural artifact of the spinning top, these represent an option for mathematics teachers to problematize in schools the local (emic) knowledge that revolves around the practices that develop in the elaboration of the spinning top, with themes of the mathematical knowledge that is traditionally practiced globally (etic) in schools, which revolve around the calculation of volumes of geometric bodies such as the cylinder and calculation of areas of cylindrical surfaces and circles (Manchego; Utria; Aroca, 2024; D'Ambrosio, 2014, 2021; Rosa; Orey; Gavarrete, 2017; Rosa *et al.* 2022).

[3] According to Levinas (1970), alterity derives from the Latin word *alter*, which is a philosophical term related to the quality of being different. It is generally taken as the philosophical principle of changing one's own perspective for that of *others*. Thus, otherness refers to the state of being of others and to diversity. It contains concepts such as difference and alterity within itself. It is important to unravel difference and alterity to begin to understand alterity and the set of meanings associated with differences.

Among the limitations of this research, it is important to note that the results are derived from the analysis and translation of a specific sociocultural practice associated with the construction of a spinning top by a particular group of participants. Therefore, it is necessary to recognize that the results of this research present limitations related both to the situated nature of the sociocultural practice studied and to the difficulties inherent in the processes of cultural translation (Rosa; Orey, 2017b, 2024b, 2024a).

Indeed, the construction of etic ethnomodels from local knowledge, knowings, and doings involves interpreting practices developed in specific sociocultural contexts through categories typical of academic mathematics, which can lead to displacements, reinterpretations, or simplifications of their meanings. For this reason, the proposed ethnomodels should not be understood as exhaustive representations of the cultural complexity associated with the spinning top nor as generalizable models to other practices or sociocultural contexts. Rather, they constitute situated interpretations that seek to establish a dialogue between different knowledge systems, recognizing both the possibilities and the limits of translation between local and academic perspectives.

In general terms, this research made it possible to identify mathematical knowledge, knowings, and doings present in the elaboration of the bottle cap spinning top and, from them, to build etic (global) ethnomodels based on concepts of integral calculus. Among the main findings, we have the identification of geometric relationships associated with the radius, diameter, center, area, and volume of the cultural artifact, as well as the development of mathematical models that make it possible to analytically represent the body of the spinning top through triple integrals and algebraic expressions. These results show that local sociocultural practices can constitute legitimate sources for the construction of ethnomodels in formal academic contexts, favoring the dialogue between emic (local) knowledge and etic (global) academic knowledge (Rosa, 2010; Rosa; Orey, 2017b, 2017a, 2024b, 2024a; Rosa *et al.* 2022).

Therefore, rather than offering results that can be generalized to any cultural or educational context, this study seeks to provide evidence of the theoretical and methodological feasibility of building ethnomodels from specific sociocultural practices and placing them in formal mathematical educational spaces, particularly in Integral Calculus university courses.

Therefore, the results of this research, supported theoretically and methodologically by the Ethnomathematics Program and ethnomodelling, offer a broader, more complete, and holistic understanding of mathematical practices, since they take into account the mathematical *knowings* and *doings* of members of different cultural groups that, originally, are not included in the mathematical curriculum content taught in schools and universities.

The dialogical context offered by ethnomodelling and the mathematical curriculum based on the perspective of ethnomathematics provide an underlying philosophy for the translation between different mathematical knowledge systems and to guarantee the development of a teaching and learning process in mathematics according to an inter and transcultural perspective (D'Ambrosio, 1985, 2002, 2021, 2019; Rosa, 2010; Rosa; Orey, 2017b, 2017a, 2024b; Rosa; Orey; Gavarrete, 2017).

Finally, and because of everything raised in this article, the questions remain open:

a) How could the results of this study be problematized in an Integral Calculus course?
b) How could the results presented in this article be problematized in the math classroom?
c) How could curriculum activities be implemented within a class plan that respects and values the knowledge, *knowings*, and *doings* of the child population in a Calculus class?

These questions can guide mathematical educators in developing curriculum activities and mathematics classes that move away from traditional teaching and learning processes and provide students with tools to develop in a transcultural, non-dogmatic way by contextualizing school mathematical knowledge in the real world.

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