

Potential and challenges in teaching systems of 1st degree equations via problem solving

Potencialidades e desafios no ensino de sistemas de equações de 1º grau via resolução de problemas

Posibilidades y retos de la enseñanza de los sistemas de ecuaciones de primer grado mediante la resolución de problemas

João Alessandro da Luz ¹ & Marcelo Carlos de Proença ²

Abstract

The aim of this article is to highlight the potential and challenges involved in developing a teaching via problem solving approach for learning systems of linear equations. Twenty-eight 8th-grade students participated in the study. The data collection tools were the students' worksheets, which were analyzed using descriptive-analytical and interpretive methods. In the first stage of instruction, students relied on a trial-and-error strategy. In the subsequent stages, we observed their understanding of the concept and algorithmic processes. Some challenges were related to difficulties with algebraic calculations and monitoring problem-solving. We concluded that the potential of the instruction was to foster the development of students' understanding of the content of systems of linear equations, thereby promoting the development of algebraic thinking.

Keywords: Mathematics Teaching. Algebra. Linear Systems. Algebraic Thinking. Problem Solving.

Resumo

O objetivo do artigo é revelar as potencialidades e desafios no desenvolvimento de uma proposta de ensino via resolução de problemas para a aprendizagem de sistemas de equações de 1º grau. Participaram 28 alunos do 8º ano do Ensino Fundamental. Os instrumentos de coleta de dados foram as folhas de atividades dos alunos, cujos registros foram analisados de forma descritivo-analítica e interpretativa. Na primeira etapa do ensino, os alunos recorreram à estratégia de tentativa e erro. Nas demais etapas, observamos a compreensão do conceito e dos processos algorítmicos. Alguns desafios foram referentes às dificuldades em cálculos algébricos e no monitoramento da resolução dos problemas. Concluimos que a potencialidade do ensino foi proporcionar a evolução da compreensão do conteúdo de sistemas de equações de 1º grau, favorecendo o desenvolvimento do pensamento algébrico.

Palavras-chave: Ensino de Matemática. Álgebra. Sistemas lineares. Pensamento algébrico. Resolução de Problemas.

Resumen

El objetivo del artículo es poner de manifiesto las posibilidades y los retos que plantea el desarrollo de una propuesta didáctica basada en la resolución de problemas para el aprendizaje de los sistemas de ecuaciones de primer grado. Participaron 28 alumnos de 8.º de primaria. Los instrumentos de recogida de datos fueron las hojas de actividades de los alumnos, cuyos registros se analizaron de forma descriptiva-analítica e interpretativa. En la primera etapa de la enseñanza, los alumnos recurrieron a la estrategia de prueba y error. En las demás etapas, observamos la comprensión del concepto y de los procesos algorítmicos. Algunos retos se referían a las dificultades en los cálculos algebraicos y en el seguimiento de la resolución de los problemas. Concluimos que el potencial de la enseñanza fue facilitar la evolución de la comprensión del contenido de los sistemas de ecuaciones de primer grado, favoreciendo el desarrollo del pensamiento algebraico.

Palabras clave: Enseñanza de Matemáticas. Álgebra. Sistemas Lineales. Pensamiento Algebraico. Resolución de Problemas.

¹ Secretaria de Educação do Paraná (Seed-PR), Campo Mourão, Paraná, Brasil. E-mail: jluz@escola.pr.gov.br

² Universidade Estadual de Maringá (UEM) , Bauru, São Paulo, Brasil. E-mail: mcproenca@uem.br

1. Introduction

The learning of algebra and, consequently, the development of algebraic thinking, is a factor of extreme importance for the world. In Brazil, the study by [Becher \(2009\)](#) showed that students had difficulties understanding and signifying situations and concepts, translating written language into algebraic language, and using algebraic formulas, properties, and techniques. In Portugal, [Morais \(2012\)](#) found students with difficulties in representation and generalization in different mathematical situations. In Turkey, [Töman e Gökburun \(2022\)](#) identified students with erroneous calculations in algebraic expressions and a low level of algebraic thinking, associated with difficulties in transitioning from arithmetic to algebra. In Australia, [Chinnappan \(2004\)](#) showed that the students in their study found obstacles to solving equations with more complex structures related to algebraic conceptual knowledge.

These difficulties students find in learning algebra (and in algebraic thinking) in Brazil and worldwide open space to studies that examine its causes and others that seek to reconstruct/mitigate this challenging situation. Therefore, the resulting gap concerns research on classroom teaching. In this sense, an important algebraic content, the focus of this article, is the teaching of systems of equations. While [Ferreira \(2013\)](#) showed that this content is taught briefly and superficially, [Battaglioli \(2008\)](#) pointed out that mathematics books prioritize algebraic calculations; in turn, [Delazeri \(2017\)](#) states that this scenario exposes students to difficulties with systems of equations while having limited understanding of the content. [Cunha Neto \(2020\)](#) affirms that this panorama is made precarious by classroom teaching that does not favor an environment to discuss judgments, concepts, and the formation of theoretical thinking, giving few conditions for students' cognitive development.

Faced with this problem, [Campos \(2019\)](#) and [Campos e Farias \(2020\)](#) guide the teaching of systems of equations permeated by problem-solving activities, thus allowing the establishment of connections between natural and algebraic language. The National Common Curriculum Base – BNCC ([BNCC, 2018](#), p. 528) establishes that the teaching of systems of equations should enable the student to “solve and elaborate everyday problems”. However, [Proença, Campelo e Santos \(2022\)](#) showed that, as far as the curriculum is concerned, the BNCC for elementary school mathematics does not suggest how to deal with problem solving in the classroom, showing more of a focus on content applications, and the authors indicated the use of a teaching approach such as that of [Proença \(2021\)](#).

This article adopts, as the basis of practice, the proposal of teaching organization via problem solving with a focus on the conceptual formation of [Proença \(2021\)](#) for the content of a system of 1st degree equations. This teaching organization is structured in four stages, namely: (i) use of the problem as a starting point; (ii) formation of the concept; (iii) definition of the content; and (iv) application to new problems.

This article is derived from the research of the first author's doctoral thesis ([Luz, 2023](#)), which dealt with linear systems in high school and indicated the need for further research on this content in elementary school, as well as the use of the teaching methodology of [Proença \(2021\)](#) to involve students in solving problems for learning mathematical content. Thus, this article is inspired by that study and its activities, in which the objective is to reveal *the potentialities and challenges in the development of a teaching proposal via problem solving for the learning of systems of 1st degree equations to students of the eighth grade of elementary school*. Thus, this article presents the theoretical framework section, followed by the methodology section, the analysis and discussion of the results, the conclusion, and the references.

2. Algebraic thinking, the teaching of algebra, and systems of equations

For the National Common Curriculum Base – BNCC ([BNCC, 2018](#), p. 270), algebra aims to develop algebraic thinking, which is fundamental for students to use mathematical models that enable the

“understanding, representation, and analysis of quantitative relations of magnitudes and mathematical situations and structures”. Thus, Ponte, Branco and Matos (2009) point out three demands of competencies and skills related to algebraic thinking: *representing* – involving reading, comprehension, writing and operation with algebraic symbols; *reasoning* – encompassing the ability to relate, generalize, deduce and understand rules; and, *solving problems and modeling situations* – based on the use of algebra in solving problems in Mathematics and other domains. In this sense, Kuhn e Lima (2021) observe that, in the view of the BNCC, algebraic thinking must enable students to formulate, employ, and interpret Mathematics in different contexts, and that pedagogical practice is of great importance for this to really happen.

On the other hand, government assessments such as the 2018 PISA (Brasil, 2021a) and the 2019 SAEB (Brasil, 2021b) indicated that Brazilian students performed poorly on mathematics tests, particularly in algebra-related content. In this context, research in Brazil and abroad corroborates the noted difficulties in teaching algebra across different contexts. In Brazil, Becher (2009) noted that students had difficulty formally representing a problem, particularly translating written language into algebraic language. Morais (2012) observed in Portugal that students have obstacles to representing and generalizing different mathematical situations. On the other hand, Töman e Gökburun (2022) and Chinnappan (2004) observed that students in Turkey and Australia, respectively, found algebraic calculations challenging.

When we deal with the teaching of systems of equations, the subject of this article, we find a framework of difficulties. Jordão (2011) noted in his research that students have difficulty interpreting problem statements and solving systems of three equations. Delazeri (2017) observed students having difficulties with systems of equations and having a limited understanding of this content. Finally, in Indonesia, Maharani (2020) showed that students had limitations in choosing steps to solve systems of equations due to the requirement for more elaborate mathematical reasoning.

To circumvent this panorama, Borralho e Barbosa (2009) and Becher (2009) suggest teaching practices that promote meaningful learning so that difficulties in algebra are reconstructed. Thus, Campos e Farias (2020) propose activities that encompass algebra through problem-solving, thereby establishing relationships and connections between natural language and algebraic language. Thus, we judge the teaching approach through problem solving as a valid strategy to be tested in order to circumvent and reconstruct the aforementioned obstacles to teaching systems of equations. We will address this approach in the following section.

3. Problem solving to teach mathematics

Proença (2018) infers that, to become a problem, a mathematical situation must cause the student to mobilize previously learned mathematical concepts, principles, and procedures that lead them toward the path of the answer. Thus, its resolution does not establish the use of rules or formulas and, if this occurs, the mathematical situation tends to have the configuration of an exercise. The author also points out four steps for solving a problem that cover the domain of cognitive knowledge, namely:

(i) *Representation*: the moment when the student understands and interprets the problem they want to solve. Thus, they use their **linguistic knowledge**, which encompasses the comprehension of the mother tongue to recognize words and relationships between subjects and objects; **semantic knowledge**, which allows identifying mathematical terms and their relationships; and **schematic knowledge**, responsible for recognizing the problem in its essence, verifying whether it is algebra, geometry, among others (Proença, 2018).

(ii) *Planning*: deals with the use of a strategy in the search for solving the problem. Thus, it involves the use of **strategic knowledge** to manage an action plan, which may include the use of drawing strategies, tables, trial and error, etc. (Proença, 2018).

(iii) *Execution*: refers to the moment of executing the proposed strategy, in order to obtain an answer to the problem. In this sense, the student is guided by their **procedural knowledge**, that is, in performing mathematical calculations and drawings, which reveals their use of logical thinking as they seek to establish spatial and quantitative relationships (Proença, 2018).

(iv) *Monitoring*: the last step includes the moment the individual checks whether the answer found is in accordance with the problem's question, as well as the act of reviewing the path taken to find the solution (Proença, 2018).

In view of what a problem is and the process for solving one, in Mathematics teaching, a mathematical situation can be introduced and problems can be addressed throughout it. One possible teaching methodology is the proposal of teaching organization via problem solving, with a focus on conceptual formation, by Proença (2018), structured in four stages for the teacher's work in the classroom.

1st stage – use of the problem as a starting point: this initial stage covers teaching *via* problem solving, in which a problem is used as a starting point for a mathematical concept/content/subject. In this sense, five actions of the Teaching-Learning of Mathematics via Problem Solving (EAMvRP), proposed by Proença (2021), are used, which we will explain below:

a) *Choice of the problem*: in this first action, carried out outside the classroom, the teacher will choose a mathematical situation that will be a possible problem, aiming for students to use their prior mathematical knowledge to solve it. For this, the chosen problem cannot request the content to be worked on in its statement, but rather allow students to relate their strategies to this new content. Therefore, the situation may be elaborated by the teacher or reworked in full from an existing situation. This enables students to build the mathematical content/concept/subject that will be taught by the teacher, favoring the establishment of relationships between the mathematical knowledge used and the new knowledge to be learned. Regarding this mathematical situation, the author indicates that it should present the possibility of more than one answer and different resolution strategies.

b) *Introduction of the problem*: in this action, already in the classroom, it is indicated that students be divided into smaller groups, which helps the sharing of ideas, knowledge, and experiences. At this moment, the teacher should encourage students to try to solve it however they want or find most convenient. It is at this moment that the proposed mathematical situation may become a problem for them.

c) *Assisting students during resolution*: for this third action, the teacher's work should be aimed at helping with mistaken interpretations, frequent doubts, and the rationality of the answer found in solving the problem. Proença (2018) states that, at this moment, the teacher should act as an observer, encourager, and guide of the learning process. Therefore, by observing what students do, the teacher should draw on the stages of problem solving and their respective knowledge to assess students throughout this process. In this sense, Mota e Lozada (2022) corroborate these points, emphasizing the importance of the teacher in helping students in problem-solving processes that encompass understanding the statement, collecting data, and searching for strategies to alleviate possible difficulties encountered by individuals.

d) *Discussion of students' strategies*: this action concerns the moment of sharing the resolutions adopted by the student groups. In this sense, it is a space that allows students to perceive and build relationships between the knowledge used along the path to solving the problem. The teacher's role in this action consists of pointing out the mistakes and obstacles encountered, leading students to synthesize what they have learned and understood.

e) *Articulation of students' strategies with the content*: for this last action, the teacher must articulate the students' strategies with the mathematical content/concept/subject that will be taught.

Proença (2018) indicates that the teacher should seek the central points of the strategies, discussing the relationships that exist with the new content.

In summary, this first stage favors the development of algebraic thinking, precisely because it allows students to mobilize prior knowledge in order to *represent* contexts in algebraic language (Ponte; Branco; Matos, 2009). This helps clarify whether students are able to build a symbolic representation. In addition, this first stage allows us to verify the demand for *reasoning*, involving the generalization of contextual numerical data into algebraic forms, which enables *solving problems* (Ponte; Branco; Matos, 2009).

2nd stage – formation of the concept: this second stage covers the conceptual approach to the mathematical content/subject to be introduced. Thus, Proença (2021) suggests that the teacher should propose activities based on the use of examples, non-examples, and the identification of characteristics related to the concept. Thus, at this stage, students will develop their *mental constructs* regarding their understanding of the conceptual aspect of the content studied. At this stage, algebraic thinking for systems of 1st degree linear equations takes up more space to be developed, since the demands of *representing* and *reasoning* end up being favored.

3rd stage – definition of the content: at this stage, the teacher's work involves presenting the definition and the algorithms related to the content addressed. In this sense, Proença (2021) emphasizes that the teacher should be concerned with establishing relationships between formal mathematical language and the language adopted in the concept formation stage. This can be done through collective debates discussing understandings of mathematical structures, arriving at syntheses of the adopted representations. At this stage, the demand to *represent* must be brought in, given the use of mathematical symbols, and the demand to *reason* is favored, since students must operate with equations and systems of equations.

4th stage – application to new problems: according to Proença (2021), in this last stage, the teacher should approach their classes by proposing new and different situations that are understood as new or possible problems. Thus, the objective will be to verify the transfer of learning of the mathematical concepts and algorithms studied, evaluating how students come to develop the four stages of problem solving and their respective knowledge. In this panorama, the mathematical situations must be contextualized by referring to different themes and areas, such as biology, science, politics, economics, social life, among others. In this last stage, the demands of *representing* and *reasoning* are required, in view of how students attribute symbols to the contexts involved and how they operate with them in the form of systems of 1st degree equations. Thus, the demand to *solve problems and model situations* highlights students' capacity to use the knowledge they have built.

4. Methodology

This is qualitative research in the form of pedagogical research. According to Souza e Kerbaux (2017), qualitative research aims to present, treat, and understand social contexts. Lankshear e Knobel (2008) state that pedagogical research is conducted through the teacher's professional practice with the aim of addressing questions, concerns, and/or problems identified by the teacher.

The participants in our research were 28 students (15 female and 13 male, all under legal age) from an 8th-grade elementary school class in a public school in a city in the Midwest of the state of Paraná, in which the teacher-researcher of this study (first author) was also the mathematics teacher. In order to preserve the identity of the participants, in the analysis of results we will name each individual according to the class diary, calling them A1, A2, A3, and so on.

In our research, the data collection instruments we adopted were the record sheets of the students' nine activities, which made up our teaching proposal. "Records that collect written data from

the participant's point of view can offer useful clarifications about their thoughts" (Lankshear; Knobel, 2008, p. 215). The data collection procedure took place throughout the development of these nine activities.

Thus, our teaching organization proposal followed the 4 Stages of Proença (2021), focusing on problem solving to develop the content of systems of 1st degree equations, also aiming at the development of algebraic thinking. We used 14 50-minute classes, between September 20, 2023 and October 10, 2023. In this sense, Chart 1 below shows Activity 1, which covered the 1st stage of **using the problem as a starting point** of our teaching organization for the content of systems of 1st degree equations:

Chart 1 – Activity 1 – stage 1 - using the problem as a starting point

Atividade 1 <u>Observações:</u> - Atividade em dupla. - Resolver da maneira que quiser (achar mais conveniente). - Caso encontre a solução, não diga a resposta em voz alta nem a fale ou mostre para outras duplas. - Proibido o uso celulares e calculadoras. Situação de Matemática Na escola de Mariana são usadas duas plataformas on-line para estudar Matemática. Para a nota deste trimestre, Mariana teve que responder um total de 12 questões provenientes dessas plataformas. Cada questão da <i>Plataforma Super Quiz</i> valia 1 ponto e a questão da <i>Plataforma Math Academy</i> valia 3 pontos. Mariana respondeu e acertou as 12 questões propostas, obtendo 28 pontos como nota final das atividades realizadas. Quantas questões de cada plataforma on-line Mariana acertou?

Source: The authors.

Through Chart 1, we see that the guidelines for Activity 1 were for students to be divided into pairs, being able to solve the proposed mathematical situation however they wanted, without using cell phones or calculators and without communicating with other pairs. This activity was carried out in 2 50-minute classes and included the actions of *introducing the problem, assisting students during the resolution, discussing students' strategies, and articulating students' strategies with the content*, proposed by Proença (2018).

Chart 2 below brings to light Activities 2 to 6, which involved the 2nd stage of **formation of the concept** of our teaching organization, which lasted 4 50-minute classes:

Chart 2 – Activities 2 to 6 – stage 2 - Formation of the concept

Atividade 2	
a) O Quadro 1 a seguir apresenta apenas equações do 1º grau com uma ou mais variáveis. QUADRO 1 I) $x + 3 = 5$ II) $3x - y = 9$ III) $a + 2b - 3,7c = \frac{5}{8}$ IV) $\sqrt{2}m + 4n + \frac{p}{7} = 10$ Quais características você observa que uma equação do 1º grau pode ter?	b) Agora, o Quadro 2 traz equação que NÃO SÃO equações do 1º grau: QUADRO 2 I) $x^2 + 9 = 12$ II) $3a - 2,6$ III) $5x^3 + 2x^2 + \sqrt{3}x = 0$ IV) $10w - \frac{8}{t} = 2$ V) $3x - 1$ Quais características você observa que uma equação do 1º grau NÃO PODE ter?
Atividade 3 Dê um exemplo: a) De uma equação do 1º grau com uma variável. b) De uma equação do 1º grau com duas variáveis. c) De uma equação do 1º grau com três variáveis. d) De uma equação que não é do 1º grau. Atividade 4 Vamos retomar o sistema de equações do 1º grau visto no problema das plataformas on-line que estudamos nas aulas anteriores $\begin{cases} q + m = 12 \\ q + 3m = 28 \end{cases}$ Quais características você observa que um sistema de equações do 1º grau pode ter? Atividade 5 Dê um exemplo: a) De um sistema de equações do 1º grau com duas equações e duas variáveis. b) De um sistema de equações do 1º grau com três equações e três variáveis. Atividade 6 Tendo em vista o que você estudou nas últimas aulas, escreva com suas palavras o que você entende por: a) Equação do 1º grau. b) Sistemas de equações do 1º grau.	

Source: Reorganized from Luz (2023, p. 101-103).

Through Chart 2, in accordance with what Proença (2021) recommends, we verified that Activities 2 and 4 sought to bring, respectively, students' comprehension and understanding regarding the characteristics of a 1st degree equation and a system of 1st degree equations. In Activities 3 and 5, the students were able to propose examples and non-examples of these concepts covered. Finally, Activity 6 guided a moment in which the participants could bring a definition (mental construct) for a 1st degree equation and for a system of 1st degree equations. Moving on, Chart 3 below shows Activities 7 and 8 of the 3rd stage of **defining the content** of our teaching organization, which lasted 4 50-minute classes:

Chart 3 – Activities 7 and 8 – Defining the content

Classificação de um Sistema de Equações do 1º Grau

A tabela a seguir apresenta as classificações, abreviaturas possíveis de um sistema linear em relação a sua solução:

Classificação	Abreviatura	Solução
Sistema Possível e Determinado	SPD	Admite uma única solução.
Sistema Possível e Indeterminado	SPI	Admite infinitas soluções.
Sistema Impossível	SI	Não admite soluções.

MÉTODOS DE RESOLUÇÃO

Método da Substituição

Atividade 7

Com o auxílio do seu professor, do professor, resolva e classifique os sistemas de equações do 1º grau a seguir, usando o método da substituição.

A) $\begin{cases} x + y = 9 \\ x + 2y = 14 \end{cases}$ B) $\begin{cases} 4x + y = 1 \\ 2x - y = -1 \end{cases}$

Agora, é a sua vez. Resolva e classifique os sistemas de equações do 1º grau abaixo usando o método da substituição. Em caso de dúvidas, peça ajuda ao professor.

C) $\begin{cases} x + y = -5 \\ x - y = -1 \end{cases}$ D) $\begin{cases} 2x + y = 21 \\ x - y = 9 \end{cases}$

Método da Adição

Atividade 8

Com o auxílio do seu professor, do professor, resolva e classifique os sistemas de equações do 1º grau a seguir, usando o método da adição.

A) $\begin{cases} x + y = 4 \\ 2x + 3y = 9 \end{cases}$ B) $\begin{cases} x + y = 10 \\ 3x - 2y = 30 \end{cases}$

Agora, é a sua vez. Resolva e classifique os sistemas de equações do 1º grau abaixo usando o método da adição. Em caso de dúvidas, peça ajuda ao professor.

C) $\begin{cases} x + 3y = 12 \\ 2x - y = -11 \end{cases}$ D) $\begin{cases} x - y = -5 \\ x + y = -7 \end{cases}$

Source: The authors.

We observe from Chart 3 that, in this 3rd stage, the teacher-researcher introduced the class by discussing the classification of a system of 1st degree equations and then worked on the resolution algorithms – the substitution method (Activity 7) and the addition method (Activity 8). Finally, Chart 4 below brings Activity 9, referring to the 4th stage of **application to new problems** of our teaching organization, which also lasted 4 50-minute classes:

Chart 4 – Activity 9 – Application to new problems

Atividade 9

Resolva as situações de matemática a seguir:

a) Antônia adora fazer compras no site da *Loja Xangai* da China. Como esse mês é aniversário de seu neto Frederico, ela quer comprar camisas e pares de meias para presentear-lo. Dessa forma, ela viu que o site da loja pesquisada vendia dois kits com os presentes que ela gostaria de dar ao seu neto:

Kit 1: Uma camisa *Pequim* e um par de meias *Hong Kong* - Total: 30 reais.

Kit 2: Duas camisas *Pequim* e três pares de meias *Hong Kong* - Total: 67 reais.

Qual o valor de uma camisa *Pequim* e de um par de meias *Hong Kong* vendidos pela *Loja Xangai*?

b) (PROVA PARANÁ 2022 – Adaptada) Em uma loja de cosméticos, estão sendo vendidos dois pacotes promocionais cujos produtos e preços estão apresentados abaixo.

2 desodorantes e 1 creme hidratante R\$ 52,50	1 desodorante e 2 cremes hidratantes R\$ 49,50
--	---

Independente do pacote, o preço unitário de cada desodorante é o mesmo, assim como o preço de cada creme hidratante. Determine os valores unitários do desodorante e do creme hidratante vendidos nesses pacotes promocionais.

c) (PROVA PARANÁ 2022 – Adaptada) Lúcia foi a uma lanchonete e comprou um pastel de queijo e uma lata de refrigerante, pagando R\$ 10,00 no total. Bernardo foi a essa mesma lanchonete e comprou para sua família 20 pastéis de queijo iguais aos que Lúcia comprou e 5 latas de refrigerante, pagando um total de R\$ 140,00. Lúcia e Bernardo pagaram o mesmo preço em cada lata de refrigerante e o mesmo valor em cada pastel. Qual foi o preço, em reais, de cada lata de refrigerante que Lúcia e Bernardo compraram nessa lanchonete?

d) Ontem Antonieta fez 13 anos e para comemorar foi ao cinema às 19h com sua melhor amiga. Lá Antonieta comprou um refrigerante de limão e uma pipoca pagando um total 18 reais. Já sua amiga comprou 3 refrigerantes de limão e uma pipoca pagando 30 reais por tudo. Qual o valor do refrigerante de limão e da pipoca no cinema que Antonieta e sua amiga foram?

Source: The authors.

In Chart 4, we found that Activity 9 has 4 mathematical situations (possible problems) equivalent to systems of 1st degree equations studied by our students during classes. Items *a* and *d* were prepared by the teacher-researcher, with item *d* containing superfluous information that is not relevant to solving the problem. Items *b* and *c*, in turn, were adapted from the 2022 Paraná Test, an assessment designed as a diagnostic evaluation applied quarterly by the Paraná State Department of Education (Seed-PR). So that the mathematical situations presented could be understood as problems, the response alternatives from the original test were suppressed. In this scenario, Activity 9 aligns with the guidelines of Proença (2021), who points out that, at this stage, the student must have contact with a range of new mathematical situations so as to transfer the mathematical concepts and algorithms they have learned in order to solve these possible problems.

Given the activities developed and the data collected on them, the data analysis was based on descriptive-analytical and interpretative analysis. According to Gil (2007), descriptive analysis allows the characteristics and relationships of a phenomenon or population to be described and analyzed. Rosenthal (2014), in turn, elucidates that interpretative analysis allows the researcher other ways of looking at the phenomena studied, building correlations and meanings.

5. Analysis and discussion of the results

To carry out the 9 proposed activities of our teaching organization, students were divided into 15 groups (13 pairs and 2 students individually – by the participating students' own choice). Chart 5 below shows students' performance in solving the problem proposed as a starting point in Activity 1:

Chart 5 – Resolution of activity 1 of stage 1

Grupos		Etapas de Resolução de problemas				Resolução	
Número	Alunos	Rep.	Plan.	Exec.	Monit.	Estratégia	SF
1	2 e 35	✓	✓	✓	✓	TAB+TE	✓
2	4 e 9	✓	✓	✓	✓	TAB+TE	✓
3	5 e 28	✓	✓	✓	✓	TE	✓
4	6 e 23	✓	✓	✓	✓	TE	✓
5	7 e 13	✓	✓	✓	✓	TE	✓
6	8 e 31	✓	✓	✓	✓	TE	✓
7	10 e 17	✓	✓	✓	✓	TE	✓
8	12 e 16	✓	✓	✓	✓	TE	✓
9	14 e 15	✓	✓	✓	✓	TE	✓
10	18	✓	✓	✓	✓	TAB+TE	✓
11	19 e 20	✓	✓	✓	✓	TAB+TE	✓
12	21	✓	✓	✓	✓	TE	✓
13	22 e 32	✓	✓	✓	✓	TE	✓
14	26 e 30	✓	✓	✓	✓	TAB+TE	✓
15	27 e 33	✓	✓	✓	✓	TE	✓

Legenda:

Rep: Representação; Plan: Planejamento; Exec: Execução; Monit: Monitoramento;
TAB: Tabela; TE: Tentativa e Erro; SF: Solução Final.

Source: The authors.

Through the chart above, we can observe that all groups succeeded in solving the proposed problem, correctly carrying out the 4 stages of problem solving (representation, planning, execution, and monitoring). In addition, of the 15 participating groups, 10 used the trial-and-error strategy, and 5 used the strategy of building a table with trial and error to reach the final solution.

The use of these strategies initially reveals the need for an evolution in the demand to **solve problems and model situations** of these students' algebraic thinking, because even though they had already studied the content of the 1st degree equation at school, no research group used the strategy of writing an equation for the proposed problem. This result was observed in the study by Sousa e Proença (2019), who investigated 7th-grade elementary school students and showed the predominance of the trial-and-error strategy in solving the proposed problems on 1st degree equation content. This may possibly be due to a teaching approach in which these students only solved exercises involving

polynomials, equations, algebraic expressions, etc., solely to apply algorithmic procedures. Thus, it may be that the students had little contact with contextualized problems in which the algebraic concept must be mobilized.

Chart 6 below brings to light students' performance in carrying out the activities of stage 2 of conceptual formation:

Chart 6 – Resolution of activities 2 to 6 of stage 2

Atividade 2		
Comando	Características do conceito	Número de grupos
a) Apontar características de uma equação do 1º grau.	Possui letras.	4
	Possui números.	4
	Possui letras e números.	6
	Tem números reais.	3
	Tem decimais/números com vírgula.	7
	Possui frações.	8
	Possui raízes/raiz quadrada.	7
	Podemos usar letras diferentes.	8
	Letras se repetem na mesma equação.	1
	Maior expoente da letra é 1.	10
b) Apontar características de uma equação que não é do 1º Grau.	Todas as equações têm sinal de igual.	11
	Não possui sinal de igual.	12
	As letras têm expoente diferente de 1.	14
	Tem letras no denominador.	13
	As letras têm expoentes 2.	5
	Não responderam.	1
Atividade 3		
Dar um exemplo	Respostas	Número de grupos
a) De uma equação do 1º grau com uma variável.	Apresentaram o exemplo corretamente.	15
	Apresentaram o exemplo de forma equivocada.	00
b) De uma equação do 1º grau com duas variáveis.	Apresentaram o exemplo corretamente.	12
	Apresentaram o exemplo de forma equivocada.	03
c) De uma equação do 1º grau com três variáveis.	Apresentaram o exemplo corretamente.	13
	Apresentaram o exemplo de forma equivocada.	02
d) De uma equação que não é do 1º grau.	Apresentaram o exemplo corretamente.	15
	Apresentaram o exemplo de forma equivocada.	00
Atividade 4		
Sistemas de Equações do 1º Grau		
Características elencadas		Número de grupos
Todas as equações são do 1º grau		6
As letras são as mesmas/se repetem nas equações		13
Possuem uma chave.		8
Apresentam letras.		5
Todas as variáveis têm expoente 1.		7
Possuem sinal de igual.		5
Tem operações de adição e subtração.		3
Atividade 5		
Dar um exemplo	Solução	Número de grupos
a) De um sistema de equações do 1º grau com duas equações e duas variáveis.	Apresentaram o exemplo corretamente.	12
	Apresentaram o exemplo parcialmente correto (faltando chaves e sinal de igualdade).	2
	Não fizeram.	1
b) De um sistema de equações do 1º grau com três equações e três variáveis.	Apresentaram o exemplo corretamente.	10
	Apresentaram o exemplo parcialmente correto (com 3 equações e 2 variáveis).	4
	Não fizeram.	1
Atividade 6		
Escrever	Respostas/ideias elencadas	Número de grupos
a) Com suas palavras o que é uma equação do 1º grau.	Tem letras	09
	Tem sinal de igual	09
	Possui valores desconhecidos.	02
	Variáveis tem expoente 1.	10
	Não responderam.	01
b) Com suas palavras o que é um sistema de equações do 1º grau.	Conjunto de equações do 1º grau.	11
	As letras se repetem nas equações.	03
	Possuem uma chave.	03
	Não responderam.	03

Source: The authors.

In Activities 2 to 6 above, we can verify that the investigated groups were able to identify characteristics and also give examples and non-examples of 1st degree equations and systems of 1st degree equations and, at the end, present a definition of the concept (mental construct) of the content

studied. This indicates a development of the demands to **represent** and **reason** of algebraic thinking, given that students were able to recognize and interpret concepts of systems of 1st degree equations, as well as generalize this concept through examples and non-examples. Conceptual formation work on the content of linear systems with 2nd-year high school students, carried out by Luz (2023) and Luz e Proença (2025), obtained similar results for the individuals surveyed, revealing the maturation of algebraic thinking regarding this content, inherent to conceptual formation. In view of this, we can point out that this work in Stage 2 of the teaching organization via problem solving of Proença (2021) showed potential to develop the formation of the concept of equations and systems of 1st degree equations, allowing students to evolve in the demands of **representing** and **reasoning**.

Chart 7 below elucidates how students performed in carrying out Activities 7 and 8 in stage 3 of defining the content:

Chart 7 – Resolution of activities 7 and 8 of stage 3

Atividade 7			
Resolver e classificar os SE usando o MS	Desempenho	NG	Dificuldades
C) $\begin{cases} x + y = -5 \\ x - y = -1 \end{cases}$	Resolução correta	14	-
	Resolução incorreta	01	Erros em cálculos algébricos.
	Não fizeram	00	-
D) $\begin{cases} 2x + y = 21 \\ x - y = 9 \end{cases}$	Resolução correta	09	-
	Resolução incorreta	03	Erros em cálculos algébricos.
	Não fizeram	03	-
Atividade 8			
Resolver e classificar os SE usando o MA	Desempenho	NG	Dificuldades
C) $\begin{cases} x + 3y = 12 \\ 2x - y = -11 \end{cases}$	Resolução correta	11	-
	Resolução incorreta	03	Erros em cálculos algébricos.
	Não fizeram	01	-
D) $\begin{cases} x - y = -5 \\ x + y = -7 \end{cases}$	Resolução correta	09	-
	Resolução incorreta	03	Erros em cálculos algébricos.
	Não fizeram	01	-

Legenda:

SE: Sistema de equações do 1º grau.; MS: Método da substituição;
MA: Método da adição; NG: Número de grupos.

Source: The authors.

Through Chart 7, we see that most groups of participating students solved the systems of 1st degree equations correctly, both by the substitution method and by the addition method, showing that they established relationships between formal language and the conceptual formation worked on in the previous stage. However, we had students who were unsuccessful due to obstacles encountered in algebraic calculations. Figure 1 below shows the difficulties of students A08 and A31 in solving Activity 8-d:

We verify in Figure 1 that students A08 and A31 used the addition method in Activity 8-d, but made a mistake when dividing the number -12 by 2, giving it a positive sign and finding the incorrect value $x=6$. Continuing, in the second equation, the students, in addition to substituting the incorrect value of x , also failed to use the equal sign in the third line of the equation, and, finally, the value found as $y=-11$ was mistaken. These obstacles in **procedural knowledge** indicate the need for an evolution in the **reasoning** demand of algebraic thinking pointed out by Ponte, Branco and Matos (2009), given the students' difficulty in understanding the rules and algorithms for the algebraic manipulation of a 1st degree equation.

A similar result occurred in the study by Silva (2014, p. 15), since, when investigating this same mathematical content with 8th-grade elementary school students, the author observed that they presented "difficulties in understanding the subject with regard to letters, terms, sign changes when there is a change of term, and the resolution process". Thus, the difficulties in Stage 3 can be understood in the sense that, although the class teacher (first author) had debated students' errors in operating with

Figura 1 – Difficulties of students A08 and A31 in activity 8-d

Handwritten student work showing two systems of linear equations. The left system is $\begin{cases} x - y = -5 \\ x + y = -7 \end{cases}$, with the student solving for $x = 6$. The right system is $\begin{cases} y = y = -5 \\ 6 - y = -6 \end{cases}$, with the student solving for $y = -11$.

Source: The authors

the algorithm involving equations, we understand that their errors in this activity could have been discussed further.

Finally, Chart 8 shows the performance of the surveyed groups in solving Activity 9 of stage 4 of application to new problems.

Chart 8 – Resolution of activity 9 of stage 4

Atividade 9	Etapas de resolução de problemas					
	REP.	PLAN.	EXEC.	MONIT.	Estratégia/procedimento adotado	SF
a) SE esperado: $\begin{cases} x + y = 30 \\ 2x + 3y = 67 \end{cases}$ Resposta esperada: Meia: 7 reais. Camisa: 23 reais.	☒ 15 X 00 NF 00	☒ 15 X 00 NF 00	☒ 10 X 05 NF 00	☒ 10 X 05 NF 00	MA: 09 MS: 05 TE: 01	☒ 10 X 05 NF 00
b) SE esperado: $\begin{cases} 2x + y = 52,50 \\ x + 2y = 49,50 \end{cases}$ Resposta esperada: Desodorante: 18,50 reais. Creme: 15,50 reais.	☒ 15 X 00 NF 00	☒ 15 X 00 NF 00	☒ 13 X 02 NF 00	☒ 13 X 02 NF 00	MA: 08 MS: 07	☒ 13 X 02 NF 00
c) SE esperado: $\begin{cases} x + y = 10 \\ 20x + 5y = 140 \end{cases}$ Resposta esperada: Refrigerante: 4 reais.	☒ 15 X 00 NF 00	☒ 15 X 00 NF 00	☒ 10 X 05 NF 00	☒ 07 X 08 NF 00	MA: 10 MS: 05	☒ 07 X 08 NF 00
d) SE esperado: $\begin{cases} x + y = 18 \\ 3x + y = 30 \end{cases}$ Resposta esperada: Refrigerante: 6 reais. Pipoca: 12 reais.	☒ 14 X 00 NF 01	☒ 14 X 00 NF 01	☒ 14 X 00 NF 01	☒ 13 X 01 NF 01	MA: 10 MS: 04 NF: 01	☒ 13 X 01 NF 01

Legenda:

SE: Sistema de equações do 1º grau. REP: Representação; PLAN: Planejamento; EXEC: Execução; MONIT: Monitoramento; MA: Método da adição; MS: Método da substituição; SF: Solução Final; ☒: Correta; X: Incorreta; NF: Não fez

Source: The authors.

When we observe Chart 8, with the resolution of Activity 9, in which students solved new problems equivalent to systems of 1st degree equations, we see that most students chose the resolution strategy using the addition method or the substitution method. When we compare this data with Chart 5, in which the groups solved Activity 1 using the problem as a starting point, mostly adopting the trial-and-error strategy, we have an indication of an evolution in the demands to **represent**, **reason**, and **solve problems** of algebraic thinking.

In addition, observing the stages of problem solving, we see that the individuals succeeded in the representation and planning stages of problem solving, with the development of **semantic**, **schematic**, and **strategic knowledge**. This was possible because we identified that they were able to understand the proposed problems, making the transition from written language to algebraic language, writing the equivalent systems of equations. In addition, they chose the appropriate resolution strategy for the systems of equations found. However, we observed that some groups had difficulties in the execution and monitoring stages along the path taken to solve the proposed problems. Figure 2 elucidates how these difficulties of students A09 and A25 occurred when solving item c of Activity 9:

Figura 2 – Difficulties of students A09 and A25 in activity 9-c

Handwritten work for Student A09 (left):

$$\begin{aligned} X &\rightarrow \text{Pastel} \\ Y &\rightarrow \text{refrigerante} \\ \begin{cases} X + Y = 20 & (\cdot 20) \\ 20X + 5Y = 140 & (\cdot 100) \end{cases} \\ \begin{array}{r} -20X - 20Y = -200 \\ 20X + 5Y = 140 \\ \hline -15Y = -60 \\ Y = \frac{-60}{-15} \\ Y = 4 \end{array} \end{aligned}$$

Handwritten work for Student A25 (right):

$$\begin{aligned} X &\rightarrow \text{Pastel} \\ Y &\rightarrow \text{refrigerante} \\ \begin{cases} X + Y = 20 \\ 20X + 5Y = 140 \end{cases} \\ \begin{array}{r} X + Y = 20 \\ 20X + 5Y = 140 \\ \hline -19Y = -120 \\ Y = \frac{-120}{-19} \\ Y \approx 6.3 \end{array} \end{aligned}$$

Source: The authors

From Figure 2 above, we verified that students A09 and A25 correctly wrote the system of 1st degree equations equivalent to the proposed problem and used the addition method strategy to solve it. Thus, the students correctly found the value of the pastel to be $y=4$ reais. However, in the calculation from the second to the third line, they made a mistake and divided 10 by -4, when the correct operation would have been the subtraction $10-4$, evidencing difficulties in the use of **procedural knowledge**. Thus, the students found $x=-2.5$, which is an inadequate value for the price of the pastel and, even so, they did not notice this fact, recording it as the value found in their final answer. Because of this, the group presented difficulties both in the execution stage and in the monitoring stage of the proposed problem.

Similar results occurred in the studies by Lima e Abreu (2025), when working with the content of quadratic equations, and Roberts e Le Roux (2019) and Luz e Proença (2025), in the teaching of systems of equations, in which they pointed out students' difficulties in operating with algebraic symbols and performing arithmetic operations. These studies indicated the need for the teacher to help their students, such as by revisiting numerical forms to help them understand the procedures involved. In our study, the difficulties in Stage 4 were not overcome by some groups, which may stem from something in the previous Stage 3, such that there should be work to review difficulties at the end of that Stage 3. Therefore, this is something our study should follow up on in future teaching proposals.

6. Conclusion

This research aimed to reveal *the potentialities and challenges in the development of a teaching proposal via problem solving for the learning of systems of 1st degree equations by eighth-grade elementary school students*. To achieve the proposed objective, we made use of Proença (2021)'s proposal of teaching organization via problem solving, based on four stages: use of the problem as a starting point, formation of the concept, definition of the content, and application to new problems.

Regarding stage 1, *Use of the Problem as a Starting Point*, we observed that all 15 groups surveyed correctly solved the proposed problem. However, they adopted a trial-and-error strategy, indicating the need to develop the demand to **solve problems and model situations** of algebraic thinking.

Regarding stage 2, *Formation of the Concept*, we found that the participants were able to correctly point out characteristics of examples and non-examples of the content studied, leading to an understanding of the mathematical form of systems of 1st degree equations, as well as of a mental construct coherent with the mathematical concept. Thus, we could observe an evolution of the demands to **represent** and **reason** in algebraic thinking.

Stage 3, *Definition of the Content*, in turn, showed that most students were successful in using the substitution and addition resolution methods for systems of 1st degree equations, indicating that they were able to establish relationships between the formal aspects of this content and the conceptual formation of stage 2. However, some groups made mistakes in algebraic calculations, indicating difficulties in applying **procedural knowledge** and in the **reasoning** demand of algebraic thinking.

Finally, in stage 4, *Application to New Problems*, we found that most of the participating groups correctly wrote the system of 1st degree equations equivalent to the proposed problems, adopting the algebraic resolution strategy (substitution method and/or addition method) as the path to be followed in search of the answer to the mathematical situation presented. This indicates an evolution of the demands to **represent** and **reason** in algebraic thinking, in contrast to what we saw in stage 1. However, as occurred in stage 3, we again had some groups with difficulties in algebraic calculations and in evaluating the final answer to the proposed problems. These results showed difficulties yet to be overcome in the use of **procedural knowledge** in the execution stage and in the attitude of reviewing this knowledge in the monitoring stage of problem solving.

However, we can point out that we achieved our objective, since, in developing the teaching proposal, our research revealed both potentialities and challenges. Regarding the potentialities, it was possible to foster advances in the comprehension, understanding, and construction of the concept of systems of 1st degree equations and the algorithmic processes involved. Thus, this culminated in the advancement of the conceptual and procedural knowledge of the students surveyed. In this sense, we had indications of the construction of the demands of algebraic thinking that are important for school development in the discipline of mathematics.

Regarding the challenges, already in Stage 1, the students did not mobilize algebraic strategies related to the use of 1st degree equations, which were previously studied content. Another challenge was that, even though procedural knowledge had been addressed in Stage 3, some groups had difficulties with this knowledge in operating with the equations in the system of equations. Still with regard to the students, another challenge was that, in Stage 4, the mobilization of procedural knowledge and the monitoring of the resolution followed did not occur as expected on the part of some groups. Thus, the demands of algebraic thinking to reason still need to be developed. Finally, we understand that the development of the teaching proposal poses challenges in two respects: we could have asked the students to try to propose an algebraic strategy at the end of Stage 1, which would have helped us understand in depth whether their knowledge of the use of equations was in fact not well formed; and we could have reviewed, with greater emphasis, from the groups that had difficulties, the errors in performing algebraic calculations and the absence of monitoring of their resolutions.

Regarding the scientific research carried out in this pedagogical context, we can point out a contribution in relation to studies that used the problem as a starting point, which is the focus on conceptual formation. Proença (2021)'s proposal of teaching organization via problem solving allows us to investigate how students build knowledge of the concept of a 1st degree linear equation, and not only procedural knowledge (as is done in the traditional way of teaching), in order to then analyze the use of this knowledge in contextualized situations. Thus, both from a pedagogical and a scientific research perspective, our study shows a contribution to the evolution and development of the mathematical concept addressed within the scope of problem solving and the development of algebraic thinking.

Finally, future studies could take these challenges faced by students and teaching into account in order to organize teaching proposals via problem solving in the view of Proença (2018, 2021), one of which would be to seek to understand whether students know how to use prior knowledge such as 1st degree equations. Another important aspect would be to incorporate a post-test type assessment some time later to measure students' understanding. If this is done, we can help improve students' performance on large-scale tests such as Prova Brasil and state tests, as well as the PISA test.

7. Acknowledgements

This work was carried out with the support of the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brasil (CAPES) – Financing Code 001.

Referências

BATTAGLIOLI, Carla dos Santos Moreno. **Sistemas de equações na segunda série do EM: um olhar sobre os livros didáticos**. 2008. f. 113. Dissertação (Mestrado em Educação) – Pontifícia Universidade Católica de São Paulo, São Paulo. Citado 1 vez na página 2.

BECHER, Ednei Luis. **Características do pensamento algébrico de alunos do 1º ano do Ensino Médio**. 2009. Dissertação (Mestrado em Ensino de Ciências e Matemática) – Universidade Luterana do Brasil, Canoas. Citado 3 vezes nas páginas 2, 3.

BORRALHO, António; BARBOSA, Elsa. Exploração de padrões e pensamento algébrico. In: PATTERNS: Multiple Perspectives and Contexts in Mathematics Education. 2009. p. 59–68. Citado 1 vez na página 3.

BRASIL. MINISTÉRIO DA EDUCAÇÃO. **Base Nacional Comum Curricular (BNCC): Educação é a Base**. Brasília: MEC, 2018. Citado 2 vezes na página 2.

BRASIL. MINISTÉRIO DA EDUCAÇÃO; INEP. **Resultados PISA 2018**. Brasília: INEP, 2021. Disponível em: http://portal.inep.gov.br/artigo/-/asset_publisher/B4AQV9zFY7Bv/content/pisa-2018-revela-baixo-desempenho-escolar-em-leitura-matematica-e-ciencias-no-brasil/21206. Acesso em: 21 out. 2021. Citado 1 vez na página 3.

BRASIL. MINISTÉRIO DA EDUCAÇÃO; INEP. **Resultados SAEB 2019**. Brasília: INEP, 2021. Disponível em: <https://www.gov.br/inep/pt-br/assuntos/noticias/saeb/desempenho-do-ensino-medio-melhora-no-saeb-2019>. Acesso em: 21 dez. 2024. Citado 1 vez na página 3.

CAMPOS, Márcia Azevedo. **Uma sequência didática para o desenvolvimento do pensamento algébrico no 6º ano do ensino fundamental**. 2019. Tese (Doutorado em Ensino, Filosofia e História das Ciências) – Universidade Federal da Bahia, Salvador. Citado 1 vez na página 2.

CAMPOS, Márcia Azevedo; FARIAS, Luiz Márcio Santos. A Educação Matemática e o ensino de álgebra na perspectiva de desenvolvimento do pensamento algébrico. **Revista Binacional Brasil-Argentina: Diálogo entre as Ciências**, v. 9, n. 1, p. 167–188, 2020. Citado 2 vezes nas páginas 2, 3.

CHINNAPPAN, Mohan. Relational knowledge and successful problem solving in algebra. **Journal of Science and Mathematics Education in Southeast Asia**, v. 27, n. 2, p. 112–134, 2004. Citado 2 vezes nas páginas 2, 3.

CUNHA NETO, Júlio Henrique da. **Organização do ensino-aprendizagem de sistema de equações no ensino médio: um experimento didático-formativo**. 2020. Tese (Doutorado em Educação) – Universidade de Uberaba, Uberaba. Citado 1 vez na página 2.

DELAZERI, Giovani Rosa. **A competência de resolução de problemas que envolvem o pensamento algébrico: um experimento no 9º ano do ensino fundamental**. 2017. Dissertação (Mestrado em Ensino de Ciências e Matemática) – Universidade Luterana do Brasil, Canoas. Citado 2 vezes nas páginas 2, 3.

FERREIRA, Ana Eliza Gonçalves. **A importância dos sistemas de equações no EM e a contribuição para a matemática e suas aplicações**. 2013. f. 94. Dissertação (Mestrado em Matemática) – Universidade Estadual de Ponta Grossa, Ponta Grossa. Citado 1 vez na página 2.

GIL, Antonio Carlos. **Como elaborar projetos de pesquisa**. 4. ed. São Paulo: Atlas, 2007. Citado 1 vez na página 9.

KUHN, Malcus Cassiano; LIMA, Eveline. Álgebra nos Anos Finais do Ensino Fundamental: reflexões a partir dos PCN e da BNCC para construção do pensamento algébrico significativo. **Revista de Ensino de Ciências e Matemática**, v. 12, n. 3, p. 1–23, 2021. Citado 1 vez na página 3.

LANKSHEAR, Colin; KNOBEL, Michele. **Pesquisa pedagógica: do projeto à implementação**. Artmed, 2008. Citado 2 vezes nas páginas 5, 6.

LIMA, Jefferson Barbosa de; ABREU, Jair Dias de. Animações Matemáticas com Manim: Explorando o Pensando Visual na Equação Quadrática. **Revemop**, v. 7, e2025019, 2025. Citado 1 vez na página 13.

LUZ, João Alessandro da. **Contribuições de uma proposta de ensino por meio da resolução de problemas para a aprendizagem de sistemas lineares no ensino médio**. 2023. Tese (Doutorado em Educação para a Ciência e a Matemática) – Universidade Estadual de Maringá, Maringá. Citado 3 vezes nas páginas 2, 7, 11.

LUZ, João Alessandro da; PROENÇA, Marcelo Carlos de. Contribuições de uma organização de ensino de sistemas lineares via resolução de problemas no 2º ano do ensino médio. **Educação Matemática em Pesquisa**, v. 27, n. 1, p. 217–246, 2025. Citado 2 vezes nas páginas 11, 13.

MORAIS, Ana Margarida Leandro. **A exploração de sequências e regularidades como suporte para o desenvolvimento do pensamento algébrico**. 2012. f. 235. Tese (Doutorado em Educação) – Instituto de Educação, Universidade de Lisboa, Lisboa. Citado 2 vezes nas páginas 2, 3.

MOTA, Felipe Miranda; LOZADA, Cláudia de Oliveira. Uma reflexão sobre o processo de avaliação na utilização da Resolução de Problemas no ensino de Matemática. **Revemop**, v. 4, e202208, 2022. Citado 1 vez na página 4.

PONTE, João Pedro da; BRANCO, Neusa; MATOS, Ana. **Álgebra no ensino básico**. 1. ed. Lisboa: Universidade de Lisboa, 2009. Citado 1 vez na página 5.

PROENÇA, Marcelo Carlos de. **Resolução de problemas: encaminhamentos para o ensino e a aprendizagem de matemática em sala de aula**. 1. ed. Maringá: EdUEM, 2018. Citado 9 vezes nas páginas 3–6, 15.

PROENÇA, Marcelo Carlos de. Resolução de Problemas: uma proposta de organização do ensino para a aprendizagem de conceitos matemáticos. **Revista de Educação Matemática**, v. 18, e021008–e021008, 2021. Citado 14 vezes nas páginas 2, 4–7, 9, 11, 13–15.

PROENÇA, Marcelo Carlos de; CAMPELO, Caleb da Silva Araújo; SANTOS, Renato Rodrigues dos. Resolução de Problemas na BNCC: reflexões para a sua inserção no currículo e no ensino de Matemática do Ensino Fundamental. **Revista de Ensino de Ciências e Matemática**, v. 3, p. 1–20, 2022. Citado 1 vez na página 2.

ROBERTS, Anthea; LE ROUX, Kate. A commognitive perspective on Grade 8 and Grade 9 learner thinking about linear equations. **Pythagoras**, v. 40, n. 1, p. 1–15, 2019. Citado 1 vez na página 13.

ROSENTHAL, Gabriele. **Pesquisa social interpretativa: uma introdução**. Edipucrs, 2014. Citado 1 vez na página 9.

SILVA, Justiniano Amarante da. **O ensino das equações do 1º grau no ensino fundamental com o uso de balanças**. Araruna, 2014. p. 38. Citado 1 vez na página 11.

SOUSA, Amanda Cristina de; PROENÇA, Marcelo Carlos de. Uma proposta de ensino de equação de 1.º grau com uma incógnita via resolução de problemas. **Revista Prática Docente**, v. 4, n. 2, p. 431–451, 2019. Citado 1 vez na página 9.

SOUZA, Kellcia Rezende; KERBAUY, Maria Teresa Miceli. Abordagem quanti-qualitativa: superação da dicotomia quantitativa-qualitativa na pesquisa em educação. **Educação e Filosofia**, v. 31, n. 61, p. 21–44, 2017. Citado 1 vez na página 5.

TÖMAN, Ufuk; GÖKBURUN, Ömer. What Was and Is Algebraic Thinking Skills at Different Education Levels? **World Journal of Education**, v. 12, n. 4, p. 8–20, 2022. Citado 2 vezes nas páginas 2, 3.

Appendix – Editorial Details

Editorial History

Received: 15/01/2026

Accepted: 22/04/2026

Published: 10/05/2026

How to cite (ABNT)

LUZ, João Alessandro da; PROENÇA, Marcelo Carlos de. Potential and challenges in teaching systems of 1st degree equations via problem solving. **Revemop**, Ouro Preto/MG, v. 8, e2026011, 2026. <https://doi.org/10.33532/revemop.e2026011>

How to cite (APA)

Luz, J. A. da., & Proença, M. C. de. (2026). Potential and challenges in teaching systems of 1st degree equations via problem solving. *Revemop*, 8, e2026011. <https://doi.org/10.33532/revemop.e2026011>

Funding

Not applicable.

Conflicts of Interest

The authors declare that there is no conflict of interest of a personal, commercial, academic, political, or financial nature regarding this article.

Authors' Contribution

Resumo/Abstract/Resumen: João Alessandro da Luz, Marcelo Carlos de Proença; **Introduction or First considerations:** João Alessandro da Luz, Marcelo Carlos de Proença; **Theoretical framework:** João Alessandro da Luz, Marcelo Carlos de Proença; **Methodology:** João Alessandro da Luz, Marcelo Carlos de Proença; **Data analysis:** João Alessandro da Luz, Marcelo Carlos de Proença; **Discussion of results:** João Alessandro da Luz, Marcelo Carlos de Proença; **Conclusion or Final considerations:** João Alessandro da Luz, Marcelo Carlos de Proença; **References:** João Alessandro da Luz, Marcelo Carlos de Proença; **Manuscript review:** João Alessandro da Luz, Marcelo Carlos de Proença; **Approval of the final published version:** João Alessandro da Luz, Marcelo Carlos de Proença.

CRediT – Contributor Role Taxonomy – <https://credit.niso.org/>.

Data Availability

Not applicable / These research data have not been published in the data repository; however, the authors are committed to sharing them if the reader is interested.

Copyright

Copyright is held by the authors, who grant **Revemop** the exclusive rights of first publication. Authors will not be remunerated for publishing papers in this journal. Authors are authorized to sign additional contracts separately, for non-exclusive distribution of the version of the work published in this journal (e.g., publication in an institutional repository or as a book chapter), with acknowledgment of authorship and initial publication in this journal. The editors of **Revemop** have the right to make textual adjustments and adapt to the publication standards.

Open Access

This article is Open Access, and authors are **not charged any fees** for submission or processing of their articles (**Article Processing Charges – APCs**). Open access is a broad international movement that aims to provide free and unrestricted online access to academic information, including publications and data. A publication is defined as 'open access' when there are no financial, legal, or technical barriers to accessing it – that is, when anyone can read, download, copy, distribute, print, research, or use it in education or any other way, subject to legal agreements.



Use License

This article is licensed under the **Creative Commons Atribuição 4.0 Internacional (CC BY 4.0)** license. This license allows you to share, copy, and redistribute the article in any medium or format. Furthermore, it allows you to adapt, remix, transform, and build upon the material, as long as due credit is given for authorship and initial publication in this journal.



Similarity Check

This article has been subjected to a similarity check using text detection software *iThenticate* from Turnitin, through the service **Similarity Check** da Crossref.



Review Process

Double-blind peer review

Reviewers

Two *ad hoc* reviewers evaluated this article and did not authorize the disclosure of their names.

Editor-in-Chief

Prof. Dr. Douglas da Silva Tinti

Universidade Federal de Ouro Preto (UFOP), Ouro Preto, Minas Gerais, Brasil.

Associate Editors

Prof. Dr. Edmilson Minoru Torisul

Universidade Federal de Ouro Preto (UFOP), Ouro Preto, Minas Gerais, Brasil.

Prof. Dr. José Fernandes da Silva

Instituto Federal de Educação, Ciências e Tecnologia de Minas Gerais (IFMG), Campus São João Evangelista, Minas Gerais, Brasil