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A Simple Application of the Concept of Even and Odd Numbers

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Abstract

In this short note is presented a simple activity to high-school level students using the concept of even and odd numbers. In this attempt the ideas of number, algorithm and programming are briefly discussed. These ideas, along with some basic concepts of physics, may serve as an auxiliary tool to help students to develop their scientific thought and either to emphasize the interdisciplinary nature of science.

Keywords

number, even and odd numbers, algorithm, programming.

1 Introduction

The idea of number is a fundamental idea of mathematics and is as old as human prehistory. There are archaelogical evidences of the use of primitive process of counting that date from several thousands of years ago. The idea of number arose because of the need of counting, measuring and labeling: counting the seasons of the year, the individuals in a group of animals, the stars, calculating profit and loss in commercial transactions, summing collected taxes, measuring the size of a plantation field, the quantity of water in a recipient, labeling the young and the old cattle, and so on. Indeed, the use and knowledge on numbers has evolved along with mankind and in a large extent it is the base of modern science and technology. Powerful devices such as computers and cell phones would not exist without them. Despite the apparent difficulties to handle with numbers, there are some facts and curiosities about them, their names and words associated with numbers that are worthy to mention and may be used to call attention on them, as for example a bicycle that means having two wheels; an octopus which means having eight feet; university semesters that come from Latin *semestris*; a famous and most widespread superstitious beliefs is that the number 13 is unlucky, in a such way that many hotels and office buildings around the world do not have a room or a floor number 13; $598 = 5^1 + 9^2 + 8^3$; $2808 = (9 \times 10 \times 11 \times 12 \times 13)/(9 + 10 + 11 + 12 + 13)$; $2999 = 2 + 999 + 999 + 999$, the number 4121 has product of its digits equal to the sum of its digits. A lot more interesting and fun things about numbers can be find out in literature [1], [2].

Formally, a number is a mathematical object belonging to a set (see [3]). The main sets or classes of numbers are the natural (N), the integers (Z), the rational (Q), the real (R) and the complex (C), and all of them are related by $N \subset Z \subset Q \subset R \subset C$. The sequence 0, 1, 2, 3, 4, 5, 6, 7, 8, …are the set of natural numbers, N. If we add to the natural their negative numbers we obtain the set of integer numbers, $Z = \{\ldots, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, \ldots\}$. If we consider the fractions into the integers we have the rational set $Q = \{\cdots -3, \ldots, -7/3 \ldots, -2, \ldots, -1, \ldots, -1/2, \ldots, 0, \ldots, 1/2, \ldots, 1, \ldots, 5/4, \ldots, 2, \ldots, 3 \ldots\}$. If we consider the rational plus the constants $\pi\,(= 3.1415\ldots)$, $\sqrt{2}$, $\sqrt{5}$, $e\,(= 2.7172\ldots)$, etc., we have the real numbers R. The complex set C includes the real plus the imaginary numbers like $2i, 3 + 5i$ and $5 - 7i$, where $i = \sqrt{-1}$.

In particular, the study of the integers Z played a special role on the development of mathematics mainly due to its historical importance, as since the ancient times many discoveries were made on the properties of these numbers. The properties of the set of integers are studied in a branch of mathematics known as Number Theory (for a overview on the history of the Number Theory see [4]) and one of the most important of these properties is the fundamental theorem of arithmetic, which states that any positive integer number greater than 1 can be represented in only one way as a product of prime numbers (apart from the order of the factors) [5]. Here, a prime number is a positive integer that is only divisible by 1 and itself, and for example, $9 = 3 \times 3; 12 = 3 \times 2 \times 2; 291 = 97 \times 3; 2310 = 11 \times 7 \times 5 \times 3 \times 2$, with 2, 3, 5, 7, 11, 97 being prime numbers.

Now, let us focus our attention on a subset of Z, that is the non-negative integers, i.e., 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ... The numbers in this subset that are exactly divided by 2 are the even positive numbers, and the others that are divisible by 2 with a remainder are the odd positive numbers [1]. The aim of this short study is to present an activity that allows teachers to handle with the fundamental ideas of number and algorithm plus the concept of programming in a short, very simple and direct way. In this attempt the definition of even and odd numbers, basic programming and also some elementary physics concepts are used to an extra activity to high-school level students.

2 Even and odd numbers

An elementary and geometrical definition of even and odd numbers comes straightforward from Fig. 1. Let the sequence of N integer numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ..., N-1 arranged within rows of squares, being that the sequence is ordered following the arrows. The numbers in the top row are the even numbers (or of even parity), which are multiples of 2, and the numbers in the bottom row are odd numbers (or of odd parity) [1]. Even numbers are that ones which are divisible by 2 (and obviously the numbers that are not divisible by 2 are odd), and this can be formally demonstrated as follows.

An integer and non negative number a can be written as

$$a = a_n \, a_{n-1} \cdots a_3 \, a_2 \, a_1 \, a_0, \tag{1}$$

where $a_0, \, a_1, \, a_2, \, a_3, \, \ldots \, , a_{n-1}, \, a_n$ are its digits. Now, let us see which

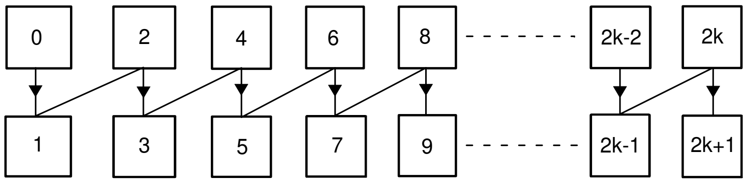


Figure 1: A schematic representation of even and odd numbers. k is an integer.

criteria this number must satisfy to be divisible by 2. The divisibility tests for a number can be evaluated by analysing the congruence properties of its digits,

$$n \equiv k (\text{mod } m), \tag{2}$$

which means that n is congruent to k (modulo m), or

$$\frac{n-k}{m} = x \tag{3}$$

where n is the base, k is the remainder and x is an integer number.

The number a can be written in terms of powers of 10 as

$$a = 10^n a_n + 10^{n-1} a_{n-1} + \cdots + 10^3 a_3 + 10^2 a_2 + 10^1 a_1 + 10^0 a_0. \tag{4}$$

For example, $245 = 10^2 \times 2 + 10^1 \times 4 + 10^0 \times 5$. Thus, the division of a by m is

$$\frac{a}{m} = \frac{10^n}{m}a_n + \frac{10^{n-1}}{m}a_{n-1} + \cdots + \frac{10^3}{m}a_3 + \frac{10^2}{m}a_2 + \frac{10^1}{m}a_1 + \frac{10^0}{m}a_0. \quad (5)$$

Thus, to verify if the integer number a is divisible by the integer number m is sufficient to verify if the sum of the right side of (5) results also in an integer number.

With the definition of congruence given in (2) we have

$$10^n \equiv 0 \pmod 2$$

$$10^{n-1} \equiv 0 \pmod 2$$

$$\vdots$$

$$10^3 \equiv 0 \pmod 2$$

$$10^2 \equiv 0 \pmod 2$$

$$10^1 \equiv 0 \pmod 2$$

$$10^0 \equiv 1 \pmod 2,$$

which means that $10^n,\ 10^{n-1},\ \ldots,\ 10^3,\ 10^2,\ 10^1$ are divisible by 2 (remainder equal to zero), and that the division of 10^0 by 2 results in a integer number with a remainder equal to 1. In other words,

$$10^n = 2x_n, \quad \text{for} \quad n \geq 1$$

$$10^n = 2x_n + 1, \quad \text{for} \quad n = 0$$

Inserting these results in (5) we obtain

$$\frac{a}{2} = x_n a_n + x_{n-1}a_{n-1} + \cdots + x_3 a_3 + x_2 a_2 + x_1 a_1 + \left(x_0 + \frac{1}{2}\right) a_0,$$

or

$$\frac{a}{2} = \left(\sum_{i=0}^{n} x_i a_i\right) + \frac{a_0}{2}. \tag{6}$$

Provided that the product of two integer numbers is integer, and the sum of two integer numbers is integer, it results that the above summation between parentheses is also integer. Thus, the number a will be divisible by 2 if its last digit a_0 is also divisible by 2. In summary, an integer number is divisible by 2 if its last digit is 0, 2, 4, 6 or 8, and therefore it is even.

At this point a discussion that may arise is if the number zero is really a even number. This question may be addressed taking into account some basic properties of the even and odd numbers, as follows:

1. one even number + one even number = one even number (as in $4 + 6 = 10$ and $16 + 20 = 36$).

   Suppose that 0 is an even number. It implies that 0 plus an even number should result in an even number. In fact, $0 + 8 = 8$ (even number) and $0 + 14 = 14$ (even number). Thus, 0 is an even number.

2. one even number + one odd number = one odd number (as in $4+7 =$

11 and $42 + 31 = 73$).

Again suppose that 0 is an even number. It implies that 0 plus an odd number should result in an odd number. In fact, $0 + 7 = 7$ (odd number) and $0 + 13 = 13$ (odd number). Thus, 0 is an even number.

3. one odd number + one odd number = one even number (as in $11+7 = 18$ and $3 + 33 = 36$).

   By hypothesis suppose that 0 is an odd number. It implies that 0 plus an odd number should result in an even number. In fact, $0 + 5 = 5$ (odd number) and $0 + 11 = 11$ (odd number). Thus, 0 is not an odd number.

4. one even number divided by 2 has no remainder (as in $48/2 = 24$ and $30/2 = 15$).

   Suppose that 0 is an even number. It implies that 0 divided by 2 should result in an integer with no remainder. In fact, $0/2 = 0$ (exactly, and note that zero is multiple of all number) leading to the conclusion that 0 is an even number. On the other side, supposing that 0 is odd $0/2$ should result in an integer with a remainder, which does not occur, again proving that 0 is an even number.

Here teachers or instructors may cite a common and popular children's joke in many countries that is the even or odd joke. They may also cite the football game, where the even or odd joke can be used to decide who will give the initial kick in the ball. Teachers or instructors may also ask to students to search for another applications of the even and odd numbers concept. May be in economics, biology, history, astronomy or daily routine?

2.1 An example of application

Matter is made of atoms and the atomic nucleus is formed by positive particles (protons) and by particles of no charge (neutrons). Both protons and neutrons are indistinctly called *nucleons* and the sum of their total quantities is the mass number, A. Thus, in a nucleus with Z protons (called its atomic number) and N neutrons we have A = Z + N nucleons. In studies of nuclear reactions an important quantity is the pairing energy. Without entering in further details of what it is, the pairing energy depends on the mass number A and on a parameter χ which is equal to 0, 1 or 2, respectively, for odd-odd, odd-even (or even-odd) or even-even nuclei. Here even and odd refer to the number of protons Z and to the number of neutrons N. For example, the nucleus of aluminium, ${}^{27}_{13}\mathrm{Al}$, has 27 nucleons (A = 27), 13 protons (Z = 13) and 14 neutrons (N = A − Z = 27 − 13 = 14), so the nucleus is odd-even (or even-odd). For the nucleus of uranium, ${}^{238}_{92}\mathrm{U}$, A = 238, Z = 92 and N = A − Z = 238 − 92 = 146, so the nucleus is even-even. And as third example, for the nucleus of silver, ${}^{108}_{47}\mathrm{Ag}$, A = 108, Z = 47 and N = A − Z = 108 − 47 = 61, so the nucleus of silver is odd-odd.

Now, a simple activity that teachers may propose for their students is to determine which nuclei are even-even, odd-even (even-odd) or odd-odd along the Periodic Table [6]. It is worth to note that, although to verify if a given number is even or odd is sufficient to see how it ends (if ends in 0, 2, 4, 6, 8 it is even, and if ends in 1, 3, 5, 7, 9 it is odd) we are interested to know within a computational routine calculation if a number is even or odd. In the next sections we will propose a solution for this problem.

3 The concept of algorithm

An algorithm is a fundamental idea of mathematics and a key concept in informatics, although its application and use do not depend on or is limited to these areas. The idea of algorithm has existed for several tens of centuries, as can be comproved by the famous algorithm formulated by Euclid (mid 4th–mid 3rd century B.C.) used for computing the greatest common divisor of two integer numbers, the largest number that divides both of them with no remainder, and by the algorithm due to Erathostenes (276–194 B.C.) used to find the prime numbers up to a certain given limit. In a simplistic manner an algorithm can be thought as a recipe to do something, and even unconsciously without knowing it we make and execute algorithms all the time along our everyday life. In a more formally way an algorithm can be defined as a specific and finite set of steps in order to perform a procedure or solving a desired problem, as outlined in Fig. 2, and is not machine or language dependent [7]. Algorithms may be represented by many different notations, such as natural languages (languages spoken and written in everyday life), pseudocodes (an informal description of an algorithm that resembles common elements of programming languages), high-level programming languages (use programming languages such as Python, JavaScript, Java, C++, etc.) and flowcharts (a pictorial representation showing the various steps of the algorithm by means of boxes). Here, we should notice that this short note is not intended to be a deep essay on the algorithms, but only to introduce the basics. For a more complete learning on algorithm, presenting its various types and features, properties, purposes, advantages, drawbacks and limitations, a specific course is desirable.

In this point teachers or instructors may discuss what an algorithm is by means of examples of daily routine. For example, an algorithm (expressed in natural language) to decide with which color of clothes will a person go out, a white or black outfit:

1. Take a shower
2. Look to the weather
3. If it is a rainy day
4. Use black clothes
5. If it is a sunny day
6. Use white clothes

The above example, but using the flowchart and pseudocode notation to represent the algorithm, is also shown in Fig. 3.

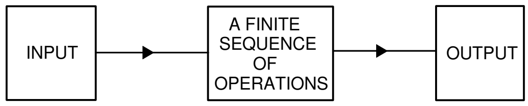
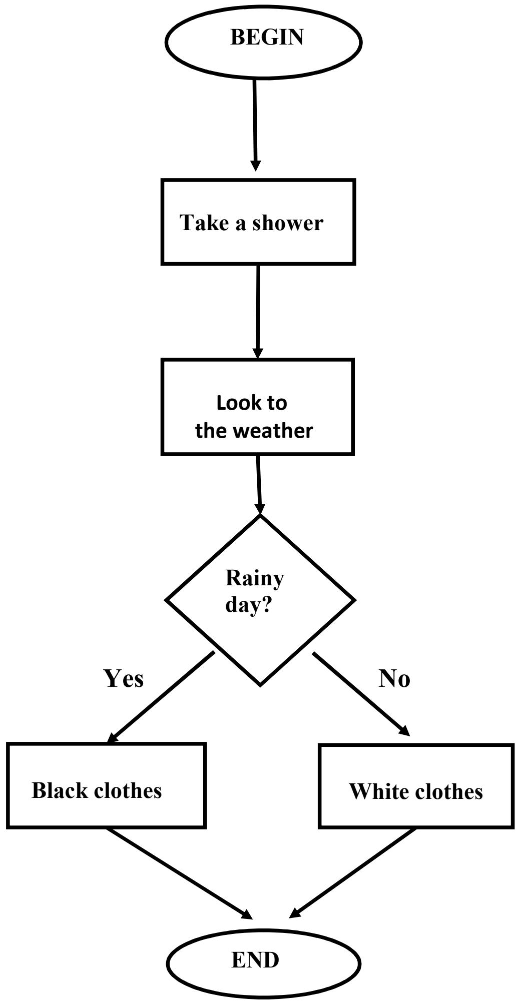


Figure 2: A simplified definition of an algorithm.

As another example, an algorithm (in natural language notation) to buy a pair of shoes:

1. Go to the shoe store
2. Ask for the aid of a seller
3. Choose a model
4. Choose a color
5. Choose your size
6. Try it
7. If it fits well
8. Pay and take it

Here teachers should emphasize that a determined problem can be solved of various different ways, in other words, more than one algorithm may be written to solve the same problem. The algorithm of the former example could also be:




```
BEGIN
  Take a shower
  Look to the weather
IF (Rainy day)
   THEN
   Use black clothes
   ELSE
   Use white clothes
END IF
END
```

Figure 3: The algorithm of first example expressed in flowchart (left) and pseudocode (right) notation.

1. Go to the shoe store
2. Choose a model
3. Choose a color
4. Choose your size
5. Pay and take it

Now, let us focus on our specific problem, that is, given a number how to know if it is even or odd. A direct and easy way to say if a number is even or odd is by successively subtracting 2 from it. If the subtractions reach in zero the number is even, if the subtractions reach in one the number is odd.

The number 9 is odd, because $9-2=7,\ 7-2=5,\ 5-2=3,\ 3-2=1$. Similarly the number 10 is even, because $10-2=8,\ 8-2=6,\ 6-2=4,\ 4-2=2,\ 2-2=0$. In Fig. 4 a summarized flow chart is used to show these successive subtractions by 2 so as to verify if the number is even or odd. In Fig. 5 a complete algorithm used to determine if a number is even or odd is shown.

4 The concept of program

A computer program can be defined as a set of instructions that directs a computer to perform a determined task [7]. Different from algorithm a program is language dependent, i.e., a program written and compiled in a certain language can not be compiled in another language. To write a program is necessary to know a programming language and users can choose among the several general-purpose available programming languages, as for example Java, PHP, Python, JavaScript, C, C++, Fortran, Objective-C, etc. A programming language is a tool to say to computer what task we want it do, or in another words, it is an implementation of an algorithm. The choice of a particular language over another depends on the type of tasks we want to perform. For example, PHP is a suitable

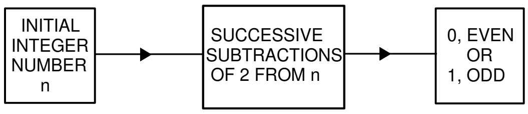


Figure 4: A summarized flow chart used to determine if a number is even or odd.

language for data base applications, Java for mobile-based applications and Fortran to handle with mathematical formulae. A question that plays a fundamental role is "What is the difference between algorithms and programs?"This question may lead to intense discussions among students and afterwards teachers may ask each of them to write the answers with their own words.

In this stage, teachers can talk about the importance of computer programming on the current days. In this sense, teachers can enumerate several appliances, comfort, conveniences and benefits that would not exist without programming:

1. No internet banking
2. No internet searching
3. No mobile
4. No apps of games
5. No vehicles

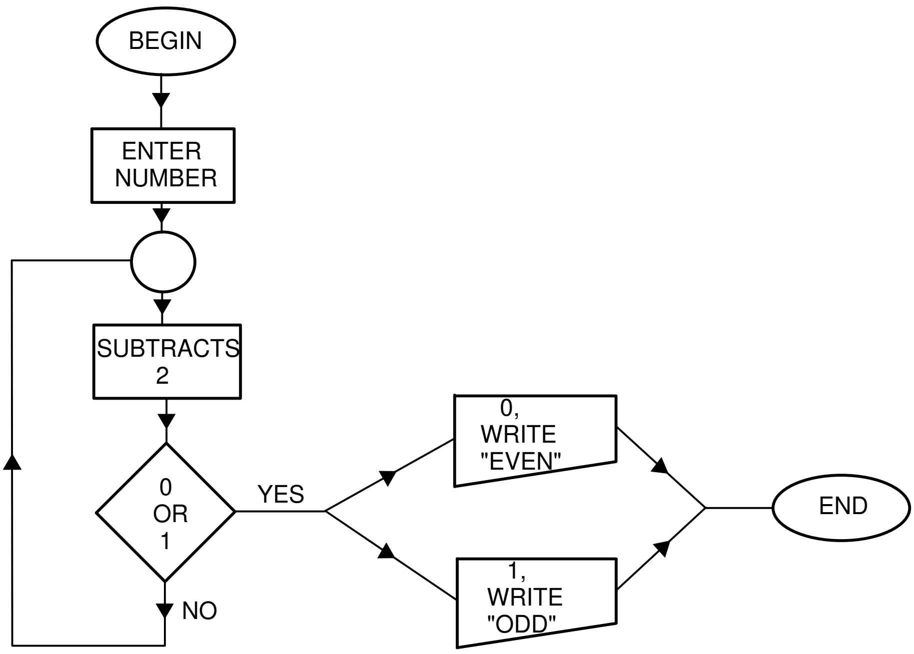


Figure 5: An algorithm used to determine if a given number is even or odd.

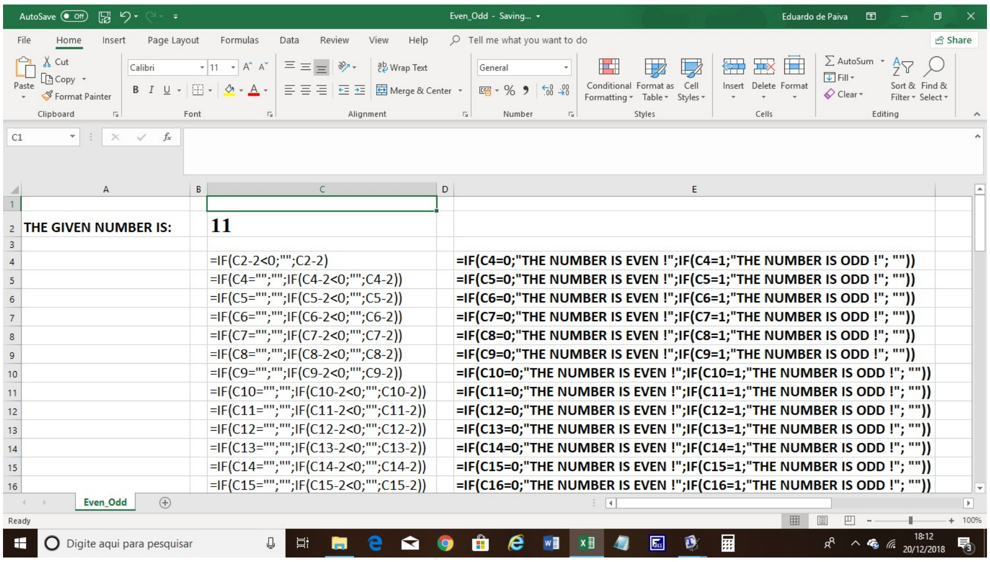


Figure 6: The equations in the excel spreadsheet used to verify if a number is even or odd.

6. No stock exchange

7. No TV

8. No radio, etc.

Although the excel spreadsheet is not exactly a programming language it will serve to our purpose. Nevertheless, students with expertise in some high level programming language may be encouraged to use it instead of excel. The algorithm depicted in Fig. 5 can be implemented in an excel spreadsheet (from now on "the program") as shown if Fig. 6. The explanations of the content of each cell are given bellow.

1. Cell $C2$: the number that we want to know if it is even or odd.

2. Cell $C4$: cell $C2-2$. But if $C2-2$ is negative nothing is written in $C4$.

3. Cell $E4$: if $C4 = 0$, the initial number is even and is written "THE NUMBER IS EVEN !". If $C4 = 1$, the initial number is odd and is written "THE NUMBER IS ODD !"
4. Cell $C5$: cell $C4 - 2$. But if $C4 - 2$ is negative nothing is written in $C5$. Also, if $C4$ is in blank nothing is written in $C5$.
5. Cell $E5$: if $C5 = 0$, the initial number is even and is written "THE NUMBER IS EVEN !". If $C5 = 1$, the initial number is odd and is written "THE NUMBER IS ODD !"
6. Cell $C6$: cell $C5 - 2$. But if $C5 - 2$ is negative nothing is written in $C6$. Also, if $C5$ is in blank nothing is written in $C6$.
7. Cell $E6$: if $C6 = 0$, the initial number is even and is written "THE NUMBER IS EVEN !". If $C6 = 1$, the initial number is odd and is written "THE NUMBER IS ODD !"
8. And so on. Note that from line 5 on there is no need to type the equations, just drag the cursor.

After typing the equations into the cells, the final appearance of the excel spreadsheet used to verify if a number is even or odd looks like the one shown in Fig. 7. It is necessary only to type the number under investigation in $C2$ and push the "arrows"or "enter"buttons. Provided that the program is properly functioning, to adapt it to the initial problem of determining if a given nucleus ${}^{A}_{Z}$X is even-even, even-odd (odd-even) or odd-odd is a quite simple task.

At this point, teachers or instructors may stimulate the students' thinking by means of some interesting questions such as:

1. Does the program function with negative numbers?
2. If not, how to modify it to accommodate the negative numbers?
3. How to modify the program in order to see that the number 0 is even and the number 1 is odd?

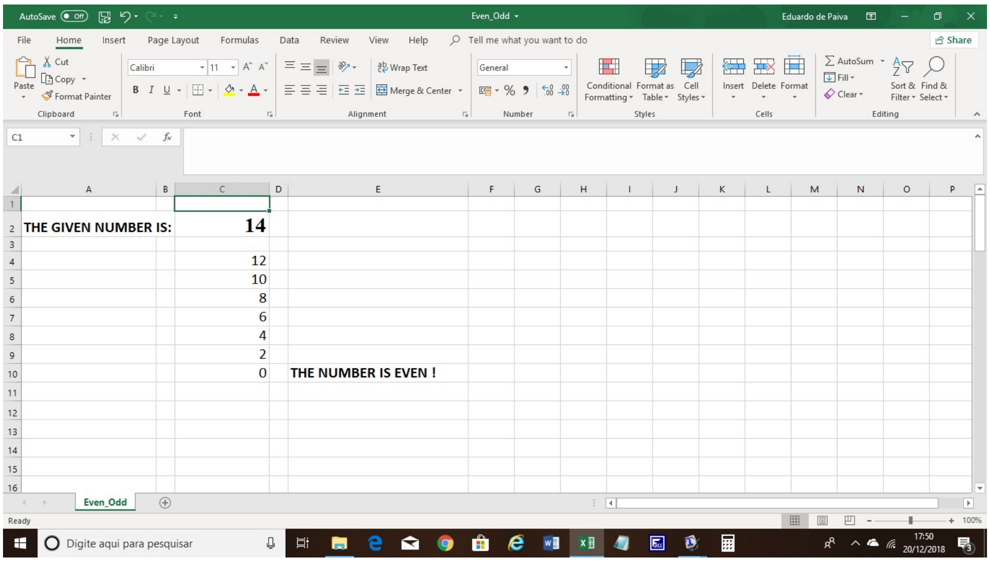


Figure 7: Final appearance of the excel spreadsheet used to verify if a number is even or odd.

4. Could we achieve the same results with successive divisions by 2?
5. If yes, how to modify the algorithm and the program in order to consider successive divisions by 2?
6. Could we achieve the same results with another arithmetic process?

5 Final remarks

In this short and simple study we propose a work to deal with the fundamental ideas of number and algorithm plus the concept of programming. In this attempt the definition of even and odd numbers, basic programming and some physics concepts are used to an extra activity to high-school level students. The discussions may start with teachers briefly talking about the concept of number and the definition of even and odd numbers. Next, teachers can guide students to the notion of algorithm, followed by an explanation of what a computer program is. Furthermore, after the proposed example of application of determining if a given atomic nucleus is even-even, even-odd or odd-odd and after all the initial discussions, teachers may used them to underline the interdisciplinary nature of science.

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